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Feature-based design for heterogeneous objects

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Abstract

Heterogeneous objects are objects composed of different constituent materials. In these objects, multiple desirable properties from different constituent materials can be synthesized into one part. In order to obtain mass applications of such heterogeneous objects, efficient and effective design methodologies for heterogeneous objects are crucial.

In this paper, we present a feature based design methodology to facilitate heterogeneous object design. Under this methodology, designers design heterogeneous objects using high-level design components that have engineering significance. These high level components are form features and material features. In this paper, we first examine the relationships between form features and material features in heterogeneous objects. We then propose three synthesized material features in accordance with our examination of these features. Based on these proposed features, we develop a feature based design methodology for heterogeneous objects. Two enabling methods for this design methodology, material heterogeneity specification within each feature and combination of these material features, are developed. A *physics (diffusion) based B-spline* method is developed to (1) allow design intent of material variation be explicitly captured by boundary conditions, (2) ensure smooth material variation across the feature volume. A novel method, *direct face neighborhood alteration*, is developed to increase the efficiency of combining heterogeneous material features.

Examples of using this feature based design methodology for heterogeneous object design, such as a prosthesis design, are presented.

Keywords: Feature based design; Heterogeneous object modeling; Solid modeling

1. Introduction

The advancement of design techniques such as the homogenization design method [2] and layered manufacturing methods [6] has made it possible to have objects composed of different constituent materials. These objects are referred to as *heterogeneous objects*. They are sometime known as functionally gradient materials (FGM). They have the ability to exhibit continuously varying composition and/or microstructure, thus producing a gradation in their properties. Such material gradation can be tailored to achieve multiple functionalities and to satisfy conflicting design requirements. These properties in general cannot be achieved by using a single material.

For example, a prosthesis using a graded interface in an orthopedic implant is shown in Fig. 1. Conventional methods of fixing an artificial bone and joining the prosthesis to the bone include total close contact of the prosthesis to the bone. However, this causes pain to the patient during weight bearing because there is micromotion of the prosthesis within the bone, and subsequently the prosthesis may even loosen in the bone. A more effective method for adhering a prosthesis to the bone is to coat it with a porous metal because new bone grows into the pores after the implantation. A graded layer of hydroxyapatite (HAp) is coated on the porous metal. It bonds to the bone physicochemically, thereby increasing the adhesion strength and the rate of binding to the bone. Therefore, porous metal with a HAp coating remedies the drawbacks of cementless prosthesis. It prevents pain to the patient caused by micromotion while walking or loosening of a prosthesis fixed without the bone cement.

Fig. 1 is a schematic structure of such an FGM interface. This FGM region is composed of porous titanium plus

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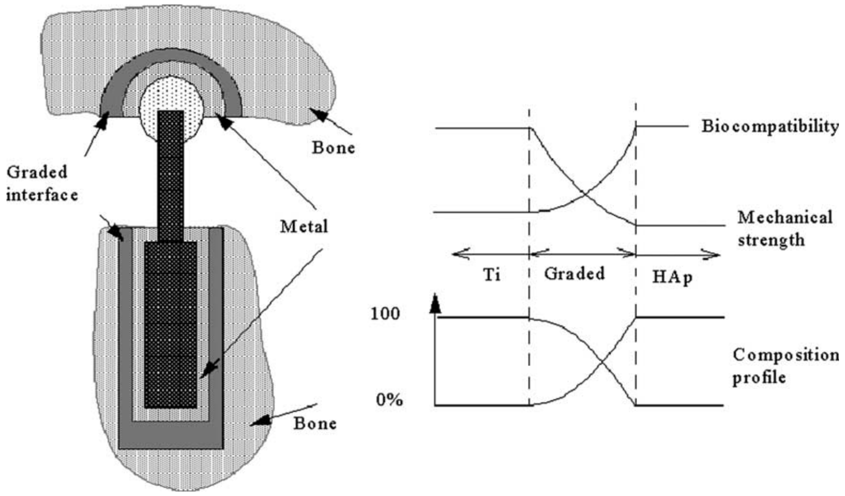

Fig. 1. Schematic structure of an FGM interface within a prosthesis.

hydroxyapatite (HAp). Ti has good mechanical toughness and HAp has good biocompatibility. Simple combination of Ti and HAp would cause bio-incompatibility and weakened strength due to their material property differences. Such material property differences are resolved in heterogeneous objects by using a mixture of Ti and HAp with varying proportions. The sharp interface between the Ti and HAp is eliminated due to a graded zone of Ti/HAp. The bending strength of the resulting material is similar to a human bone.

As evidenced in this example, many applications based on the concept of functionally gradient materials can be developed to exploit multiple desirable material properties from different materials. In order to obtain mass applications of such heterogeneous objects, efficient and effective design methodologies for heterogeneous objects are crucial. Existing design methodologies are highly sophisticated, but they are inefficient and even infeasible for handling heterogeneous objects. These methodologies are primarily developed for the production of homogeneous objects of which there is only one homogeneous material. The introduction of material variation throughout the objects adds a new dimension to the problem.

The state-of-the-art research on heterogeneous object modeling has been primarily focusing on representation schemes for heterogeneous objects. Currently, there is only limited means available to obtain heterogeneous object model. They are ineffective for heterogeneous object design in that these heterogeneous object models do not explicitly capture design intent and they do not support iterative design processes.

To address these issues, in this research we propose the use of features to facilitate the design of heteorgeneous objects. In the example of the prosthesis design, the choices of materials (Ti and HAp) and the graded interface are based on a designer's experience and these choices have specific physical reasons. Ti has good mechanical strength. HAp is a strorage form of Calcium and Phosporus in the bone and it has good bio-compatibility. We provide a design tool that allows designers to design such heterogeneous objects with a high level design component—features. Under this framework, a designer can incorporate the design intuition into the design process. Instead of specifying material composition for each spatial location within the object, the users can choose a feature (a graded interface in this example) and the desired material properties at the desired locations when designing the object.

The remainder of this paper is organized as follows. In Section 2, we present a review of the existing research relating to the design of heterogeneous objects. In Section 3, a general methodology—feature based design (FBD) for heterogeneous objects—is presented. Sections 4 and 5 present two enabling component techniques for a feature based design method, material heterogeneity modeling within each material feature and constructive feature operations for combining material features. Section 6 presents the implementation and examples of the feature based design methodology, including the feature based design process for the prosthesis design. Finally, Section 7 summarizes this paper.

2. Literature review

Many representation schemes have been developed to represent solids. Manifold solids and *R*-sets were first proposed to represent solid model [8,25]. Radial-edge data structure is another data structure for modeling non-manifold solid [32]. For conventional feature modeling, the usage of non-manifold structure was first proposed by Pratt [19]. Selected Geometric Complex (SGC) is a non-regularized non-homogeneous point set represented through enumeration as union of mutually disjoint connected open cells [26]. Constructive Non-Regularized Geometry (CNRG) was also proposed to support dimensionally non-homogeneous, non-closed point sets with internal structures [27]. Middleditch et al. presented mathematics and formal specification for the mixed dimensional cellular geometric modeling [16].

Current research on heterogeneous objects has led to many representation schemes for heterogeneous object modeling. Kumar and Dutta proposed *R-m* sets be used for representing heterogeneous objects [12]. Jackson et al. proposed another modeling approach based on subdividing the solid model into sub-regions and associating the analytical composition blending function with each region [10,11]. Park et al. presented a volumetric texturing approach for modeling heterogeneous objects [17]. Biswas et al. proposed a distance field based approach for heterogeneous object modeling, in which the space is parametrized by distance to the geometry boundaries [4]. Huang and Fadel employed a Bezier basis function for optimizing material heterogeneity in flywheel [9]. Dutta [29] and Tan [30] presented constructive approaches for heterogeneous object modeling. Kumar and Dutta presented a trivial fiber-bundle based model to represent several attributes of an object along with the geometry [13]. Pasko et al. used a functional representation (Frep) to constructively model object geometry and properties of arbitrary nature [18].

Even though existing representation schemes for heterogeneous objects provide means to represent heterogeneous objects, they do not necessarily support the heterogeneous object design process. The current methods for specifying material composition face a trade-off between the model coverage and operation convenience [23]. These methods only provide a low level description of geometry and material composition within the objects. The characteristics of material variation are not explicitly captured. The resulting material properties are often not available for designers. These methods do not provide effective tools for designers to create and edit the heterogeneous object model.

In the domain of homogeneous objects, feature methodologies have been extensively researched to facilitate their design and fabrication. Features were initially proposed to automate the link between design and NC path generation [7]. Since then, feature techniques have been widely and successfully used in CAD/CAM systems. Feature-based design expedites the design process and feature recognition facilitates the fabrication process planning. A feature-based product model also simplifies the assembly, inspection planning, and other downstream applications [28].

In order to facilitate the design and fabrication of heterogeneous objects, Qian and Dutta proposed the usage of feature methodologies for the design and layered manufacturing of heterogeneous objects [20,21]. This paper will present some of the results. Liu et al. also presented their feature based design approach to achieve local control of material properties, in which they used the Laplace equation to create material composition blending [14].

3. Feature based design for heterogeneous objects

In this section, we examine the relationships between form features and material features in heterogeneous objects. We synthesize form features and material features and we propose a constructive feature based design method for heterogeneous objects.

3.1. Features

Feature techniques, traditionally, have only been focusing on the geometry, i.e. form features. Because of the nature of material variation in heterogeneous objects, we shall examine features not only in terms of the geometry but also in terms of the material composition in a part.

In order to mathematically represent the features, we first define some notations. A part, $P(G,M)$, is defined as a product space, where G is the geometry and M is the material space.

3.1.1. Form feature

Form feature is a specific geometric shape, which carries engineering significance, such as a hole and a slot. A form feature can be either a volume feature or a surface feature. In this paper, we focus on volume features.

As with homogeneous objects, a form feature in heterogeneous objects is a specific shape within a part regardless of the material composition variation. In order to distinguish form features from material features, we note two necessary conditions to the definitions of form features. First, *the shape of the volume must correspond to some specific engineering meaning*. For example, form features such as a hole or a groove, have specific geometric shapes and engineering significance. Second, *such a shape should contribute to the formation of the boundary of the final part geometry*. That is to say, during the part creation process, the evolving part geometries should be different before and after the introduction of the form features. We note the part geometry as G_i before the form feature FF_{i+1} is introduced to the part. Then the second necessary condition for form features can be represented as: $\mathrm{FF}_{i+1} - G_i \neq \emptyset$. This will be further explained in Section 3.1.3.

For example, in Fig. 2, the heterogeneous object has three form features: a block, a hole, and a boss. They each represent a particular geometric shape. If we disregard the material variation in the object, these three form features create the final geometry of the object. In the two FGM regions, FGM1 is a form feature while FGM2 is not. FGM2 does not satisfy the second condition of form feature, i.e. FGM2, as a shape, does not contribute to the boundary of final part geometry.

3.1.2. Material feature

Before we present the definition of material features, we first examine material variation in heterogeneous objects.

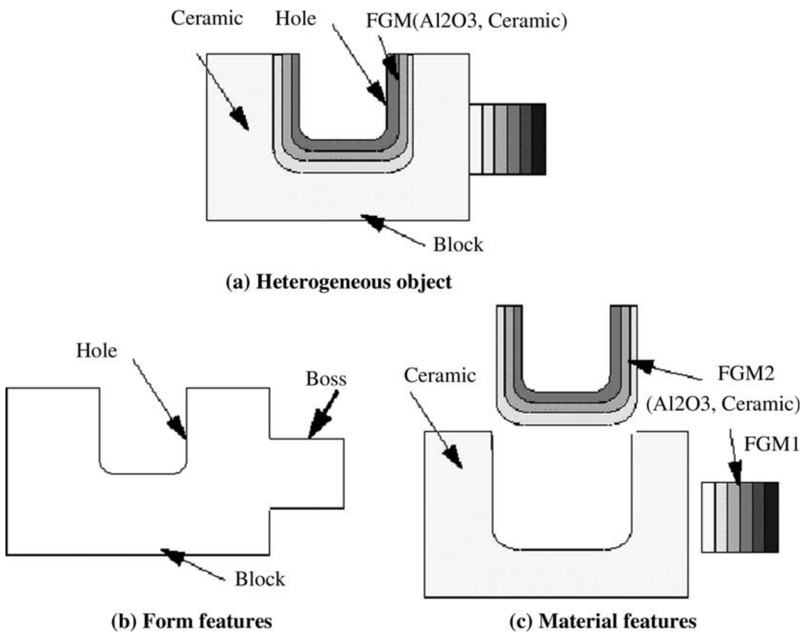

Fig. 2. Features in a heterogeneous object.

Heterogeneous materials arise in materially optimized structures where the material composition and distribution are optimized to maximize the desired performance measures. They provide a smooth transition among different materials. The material variation usually correspond to some particular functionality and design intent. They can be explicitly captured by a material volume, formally a material feature. Such a material volume can be represented in many different ways, e.g. a swept material volume [21] (See Fig. 3) or a B-spline material volume [23].

Swept material volume. Swept material volume is composed of a cross-section and a path. The material could vary along the cross-section and/or along the path.

Conventionally, a swept volume S (Fig. 3) is defined by sweeping a surface $r(u,v)$ along path $p(w)$. If ' $*$ ' denotes sweep, the swept material primitive (SM) (Fig. 3) could be written as

$$\mathrm{SM}(S,M) = (r(u,v)^{*}p(w), m(u,v,w))$$

where S represents the geometric volume, M represents the material composition, and material variation function $m(u,v,w)$ represents the material changes along the cross-section $r(u,v)$ and path $p(w)$. The material variation function could be any user-defined function (e.g. constant, linear, step function, parabolic, exponential, etc.) [15]. The function $m(u,v,w)$ would enable full three dimensional material variation. In practice, material variation typically would happen only along one of the u,v,w directions, denoted as $m(u)$, $m(v)$ and $m(w)$.

B-spline material volume

A more general representation of material volumes can be represented in a B-spline solid. Section 4 gives a detailed description of such a representation and the corresponding modeling method.

A material feature is a region with some particular material composition variation and this material variation function is different from the neighboring volume's material functions. Such material composition variation is associated with some engineering significance, such as erosion protection, thermal balance, and biocompatibility.

A material feature is an enriched material volume. The relationship between a material feature and the material volume is similar to the relationship between a form feature and the geometric volume. The features contain engineering relevance while the volumes do not. Material features can be represented as a pair, $\mathrm{MF}(g,m)$, where m has certain characteristics in the region g and is different from the material function elsewhere.

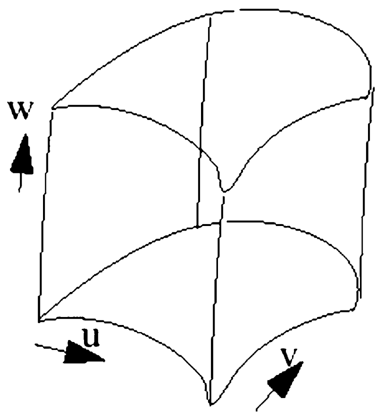

Fig. 3. Swept material volume.

In this paper, when material functions are equal to each other for two regions (g_1, m_1) and (g_2, m_2), $m_1 \equiv m_2$, it means: (1) there is a C^∞ function [13] $m(x)$ for $x \in g$, $g = g_1 \cup g_2$; (2) $m = m_1$ for $x \in g_1$; and (3) $m = m_2$ for $x \in g_2$.

The sample part in Fig. 2 has three material features: two FGM (Al2O3 and Ceramic) regions and one ceramic region.

3.1.3. Observations on form features and material features of heterogeneous objects

Next issue to be examined is the relationships between form features and material features.

Since we will define feature operations based on these features, it is important to determine what are the critical characteristics of these features. In the course of our investigation, we have observed a number of significant points regarding to the nature of these features.

Observation 1. Material features MF(g, m) form a partition of the part P. That is,

$$P = \bigcup_i \mathrm{MF}$$

$$\bar{G} = g_1|g_2|\cdots|g$$

Note, $\bar{G}$ is defined as a closure in 3d manifold, and '|' is a gluing operation.

Fig. 4 shows the partition of the part geometry by material features. In the left is a complete geometry of the part shown in Fig. 2. In the right is a partition of the part volume. Each sub-volume in the partition corresponds to one material feature in Fig. 2.

Observation 2. Form features form the geometry of a part volume

$$\bar{G} = \prod_j \mathrm{FF}_j$$

The symbol $\prod$ refers to the form feature operations, i.e. either an addition or a subtractive operation.

Fig. 2b shows how form features form the part volume. Three features are added one by one and lead to the final part geometry.

Observation 3. The geometry volumes in form feature volumes and the material volumes in material features need not to be identical.

In order to examine the relationships between form features and material features, we note the geometric volume of material feature MF_i as $g(\mathrm{MF}_i)$, its operation with FF_j as $g(\mathrm{MF}_i)\otimes\mathrm{FF}_j$. It can be simplified as $\mathrm{MF}_i\otimes\mathrm{FF}_j$. The geometry of form features and material features have one of the following relationships (Fig. 5):

- MF and FF have identical geometric volumes (*identical*)

  $\mathrm{MF}_i - \mathrm{FF}_j = \mathrm{FF}_j - \mathrm{MF}_i = \varnothing$

- MF belongs to FF or FF belongs to MF *(belonging)*

  $\mathrm{MF} \subset \mathrm{FF}$ or $\mathrm{FF} \subset \mathrm{MF}$

- MF and FF share some subvolume (*sharing*)

  $\mathrm{FF}_i \cap \mathrm{MF}_j \neq \varnothing$

- MF and FF are disjointed (*disjointed*)

  $\mathrm{FF}_i \cap \mathrm{MF}_j = \varnothing$

The above observations reveal that material features describe the part's interior material composition and form features describe the part's exterior geometric shape.

Even though using form features alone or material features alone may be able to construct the design model, using each type of features alone is not sufficient to support the design process. Using form feature alone, no proper partition of the part volume is obtained for modeling material variation. Using material features alone, the design intent of the geometric features is not captured. Often times both form features and material features are necessary representations of the design intents. Therefore, feature based design for heterogeneous object needs to include both geometric and material features.

3.2. Synthesized features for feature based constructive design

3.2.1. Synthesized features and semantics definition

With our understanding of the relationships between the form features and material features, we can now proceed to the synthesis of form feature and material feature operations.

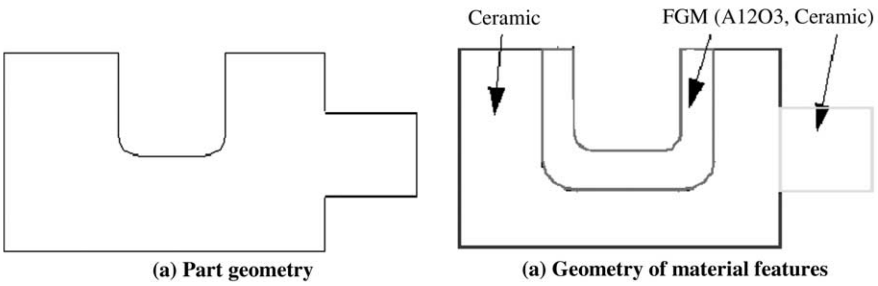

Fig. 4. Material features partitions part volume.

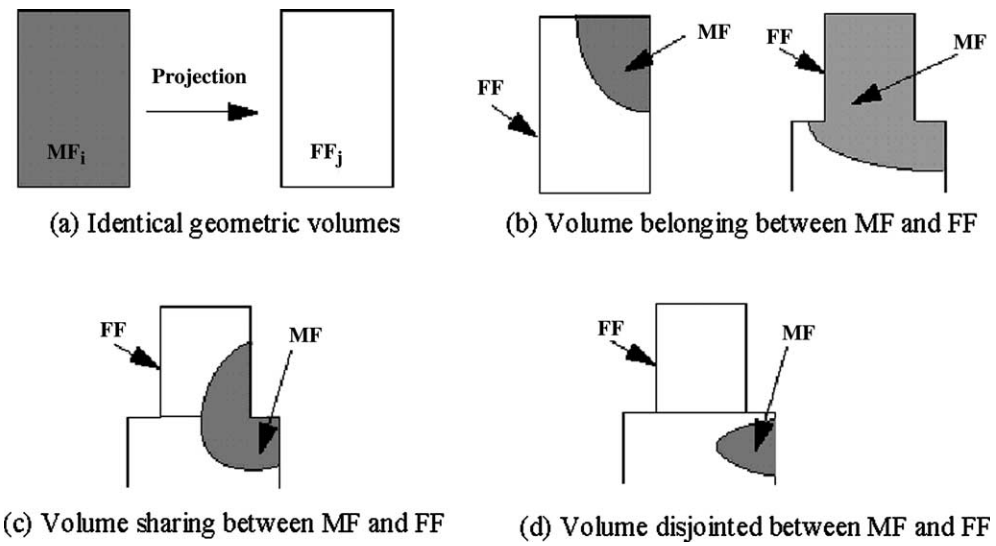

Fig. 5. Relationships between form features and material features.

In STEP [31], the volume features for homogeneous objects are classified as additive and subtractive features. In consistency with form feature classification in the STEP and the observed feature properties in heterogeneous objects, we propose the following feature operations in the context of heterogeneous object design: additive material feature, subtractive material feature and partition material feature (Fig. 6). In responding to additive and subtractive features in STEP, we propose additive and subtractive material features. In responding to the partition properties (Observation 1) of material features in heterogeneous objects, we propose partition material features. This classification is based on the modeling operation's impact on geometry.

Before we present the details of the semantics definition for each feature operation, we define some terms. For an object or a region $A(g,m), m(A)$ gives the material information m, $p(A)$ is the priority of the materials and it is useful when different materials are interacting with each other.

As noted before, '|' is the aggregate/gluing operation. '$|^*$' is the *regularized gluing* operation. For each face, if material functions over the face's two adjacent regions are equal, the face shall be eliminated. That is, $(g_1,m_1)|^*(g_2,m_2) = (g_1 \cup^* g_2, m_{12})$ when material function equality conditions are satisfied.

The three generic (synthesized) feature operations can be defined respectively as:

1. Additive material feature

$$(g_1,m_1)+(g_2,m_2)$$
$$= (g_1 - g_2, m_1)|^*(g_2 - g_1, m_2)|^*(g_1 \cap g_2, m_1 \otimes m_2)$$

2. Subtractive material feature

$$(g_1,m_1)-(g_2,m_2) = (g_1 - g_2, m_1)$$

3. Partition material feature

$$(g_1,m_1)/(g_2,m_2) = (g_1 - g_2, m_1)|^*(g_1 \cap g_2, m_1 \otimes m_2)$$

Fig. 7 lists the three types of features and their semantics. Clearly, the resultant part C of two features A and B, $C = A \otimes B$, depends on the feature type (operation), and each region's materials and the priority tag.

To resolve the material composition ambiguity over the intersection region, we introduce the material priority tag p, to each material volume. That is,

$$m_1 \otimes m_2 = \begin{cases} m_1, & \text{if } p_1 > p_2 \\ m_2, & \text{if } p_1 < p_2 \\ m_1 \oplus m_2, & \text{if } p_1 = p_2 \end{cases}$$

Note, here $m_1 \oplus m_2$ is a user defined function. It could be $a \cdot m_1 + (1-a) \cdot m_2$, $a \in (0,1)$, or any other form. $m_1 \oplus m_2$ has been particularly useful for applications like modeling doping and implanting, where material volume is 'contaminated' by some exotic materials.

How material composition change during the synthesized feature operation is referred to as *material operation semantics*.

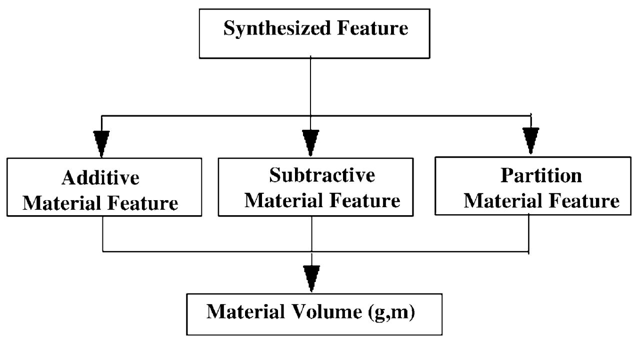

Fig. 6. A proposal for feature classification in heterogeneous objects.

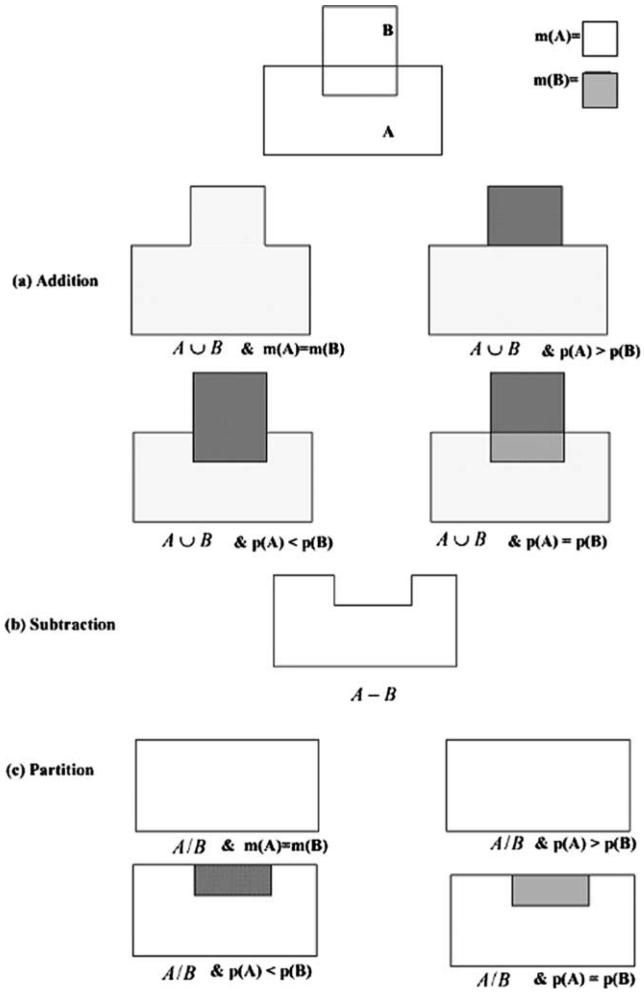

Fig. 7. Generic feature operations for heterogeneous objects.

The partition feature functions the same as additive features over the intersection region ($g_1 \cap g_2$), but it is not applicable to the region outside of g_1. This partition feature is used extensively for heterogeneous object modeling when material functions are imposed on a given geometry domain.

These synthesized features support both form feature and material feature operations. It associates each material volume with one geometric/material operator. They preclude redundant definitions of the geometry in both form features and material features. The four types of relationships between the geometric volumes of form features and material features can be fully manifested by the synthesized features in a constructive approach. In this approach, the building blocks are the synthesized features. The designer has two choices: either use the default materials to model form features and then partition the part volume with specific material composition functions, or glue a set of material feature volumes.

These synthesized feature operations can be used to design heterogeneous objects or simulate manufacturing processes involving different materials. The three features provide a generic tool for heterogeneous object modeling. Many existing design/fabrication automation tools for heterogeneous objects processing are dedicated tools and they can be directly derived from these three synthesized features. For example, the feature operation semantics used in design by composition for layered manufacturing [3] and MEMS fabrication process simulation [5] can all be derived from the synthesized features [20].

Based on these synthesized feature operations, a feature based design methodology has been developed for heterogeneous object design [20]. In our implementation, we adopted the *R-m* set as the working representation scheme for heterogeneous objects, even though other representation schemes can also be used to represent the semantic features defined above. That is, in our implementation, an *R-m* set (g, m) is used as the building block for the constructive design. A compound feature (building block), consisting of more than one *R-m* set, can also be defined, i.e. a finite collection of *R-m* sets, $(g_1, m_1), (g_2, m_2), \ldots, (g_n, m_n)$, each consisting of a material volume. To support a constructive design of heterogeneous objects, we extended the radial-edge graph to represent the geometry of heterogeneous objects [24]. In this extended data structure, each region has its material composition representation and each face use has neighborhood information, which contains a pointer pointing to material representation. Such a methodology needs two enabling component techniques: how to define material composition within each material volume and how to combine material volumes. These two enabling techniques are presented in Sections 4 and 5.

4. Material heterogeneity modeling based on design intent

In order to use synthetic material features as a building block for feature based design, we need a method to define material heterogeneity within each feature volume that can represent design intents. During the iterative design editing process, these design intents need to be preserved. In this section, we present a method in which designer specify material variation within a feature volume through designers' intuition and experience.

We use a B-spline tensor solid to represent a synthesized material feature volume. We use a virtual diffusion process to create material heterogeneity profile. The design intents are represented as a set of constraints. The detailed mathematical formulation is available in Refs. [20,22,23]. Solving diffusion equations under these constraints will automatically ensure the material variation is smooth within each material volume. In order to make this paper self-contained, we briefly present the material heterogeneity modeling method here, with emphasis on how design intents can be represented and preserved within a feature during the editing process.

4.1. B-spline tensor solid representation for material features

In order to represent freeform geometry and arbitrary material variation within a synthesized material feature, we use a B-spline tensor solid as a representation. For each point (u, v, w) in the parametric domain of a tensor product B_spline volume V, there is a corresponding point $V(u, v, w)$ at Cartesian coordinates (x, y, z) with material composition M, noted as (x, y, z, M). We define such a B-spline volume as:

$$V(u,v,w) = \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} N_{i,p}(u)N_{j,q}(v)N_{k,r}(w)P_{i,j,k} \tag{1}$$

where $P_{i,j,k} = (x_{i,j,k}, y_{i,j,k}, z_{i,j,k}, M_{i,j,k})$ are control points for the heterogeneous solid volume. $N_{i,p}, N_{j,q}$ and $N_{k,r}$ are the pth-degree, qth-degree and rth-degree B-spline functions defined in the direction of u, v, w, respectively. We can also have the B-spline representation for material properties:

$$E(u,v,w) = \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} N_{i,p}(u)N_{j,q}(v)N_{k,r}(w)E_{i,j,k} \tag{2}$$

where $E_{i,j,k}$ is material property at each control point. It can be obtained according to the volume fractions at each point. The relationship between material properties and the composition has been extensively studied [15]. For example, Eqs. (3) and (4) give the approximate relationships of thermal conductivities and mechanical strengths versus material composition. M_a, M_b are volume fractions of two composite materials at each point. λ_a, λ_b are thermal conductivities and S_a, S_b are strengths for two materials a and b.

$$\lambda = \lambda_a M_a + \lambda_b M_b + M_a M_b \frac{\lambda_a - \lambda_b}{3/(\lambda_b/\lambda_a - 1) + M_a} \tag{3}$$

$$S = S_a \cdot M_a + S_b \cdot M_b \tag{4}$$

4.2. Diffusion based heterogeneity creation

In this section, we describe how diffusion process generates different material composition profiles. Diffusion is a common physical process for the formation of material heterogeneity such as in integrated circuit fabrication, in biological mass transport, and in the drug delivery from a polymer.

The mathematical modeling of controlled material composition in these processes is based on the Fick's laws of diffusion. Applying Fick's laws and using the divergence theorem, we have the following equation for a diffusion process, in which M represents the material concentration (volume fraction for our purpose), Q is material generation source and D_{ij} is the diffusivity.

$$\frac{\mathrm{d}M}{\mathrm{d}t} = Q + \frac{\partial}{\partial x_i}\left(D_{ij} \cdot \frac{\partial M}{\partial x_j}\right) \tag{5}$$

By the B-spline finite element approximation, we have

$$KM = \vec{B} - \vec{S} \tag{6}$$

where

$$\underset{N\times N}{K_e} = \left[k \int_{\Omega_e} \left(\frac{\partial N_i}{\partial x}\cdot\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\cdot\frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z}\cdot\frac{\partial N_j}{\partial z}\right)\mathrm{d}\Omega\right] \tag{7}$$

$$\underset{N\times I}{\vec{B}_e} = \left[\int_{V_e} N^m Q \,\mathrm{d}V\right], \qquad \underset{N\times I}{\vec{S}_e} = \left[\int_{\Gamma_e} N^m q_n \,\mathrm{d}\Gamma\right] \tag{8}$$

With function Q and q interpolated in terms of its nodal values, we have $\vec{B}_{eN\times I} = [\int_{\Omega_e} N_i^m N_j^m \,\mathrm{d}\Omega]Q_j$, and $\vec{S}_{eN\times I} = [\int_{\Gamma_e} N_i^m N_j^m \,\mathrm{d}\Gamma]q_j$. K_e is the element stiffness matrix, and $\vec{B}_e$ is the element body force and $\vec{S}_e$ is the element surface force.

4.3. Design intent (constraints) based heterogeneity manipulation

The above formulation has provided a methodology to calculate the material composition for a diffusion process. It can be generalized for manipulating material composition of B-spline heterogeneous solid objects by imposing constraints, i.e. boundary conditions. These constraints are representation of design intent.

The constraints that are imposed on the B-solid include the heterogeneity information on the boundary, or the heterogeneity at specific location (u, v, w), or any other type of constraints that can be transformed into a set of equations.

Here, we consider a set of linear constraints:

$$A \cdot M = E \tag{9}$$

To accommodate the constraints in Eq. (9), solution methods generally transform this to an unconstrained system: $\min\|(1/2)(\bar{M}^T\bar{K}\bar{M} - \bar{M}^T\bar{B}\|$, in which solutions $\bar{M}$, when transformed back to M, are guaranteed to satisfy the constraints. The unconstrained system is at a minimum when its derivatives are 0, thus we are led to solve the system $\bar{K}\bar{M} = \bar{B}$. Specifically, we introduce a Lagrange multiplier for each constraint row A_i, and we then minimize the unconstrained $\min_p\|(1/2)(M^TKM - M^TB + (AM - F)G)\|$. Differentiating with respect to M leads to the augmented system:

$$\begin{bmatrix} K & A^T \\ A & 0 \end{bmatrix} \cdot \begin{bmatrix} M \\ G \end{bmatrix} = \begin{bmatrix} B \\ F \end{bmatrix} \tag{10}$$

Solving the above linear equations leads to the solution to the constrained system. Note, in this paper, for the sake of saving computational time, only linear constraints are considered.

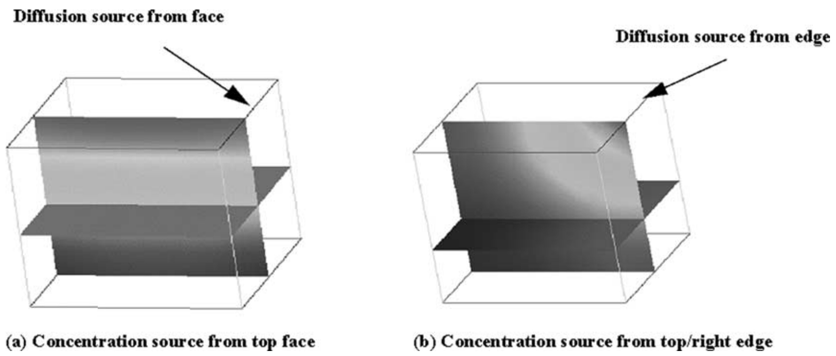

Fig. 8. Material variation created by a diffusion process.

However, it is not difficult to generalize the method to accommodate non-linear constraints by enforcing Lagrange multiplier techniques.

Two diffusion processes are shown in Fig. 8, one with concentration source from top face, one with concentration source from top/right edge. We obtained the heterogeneous objects by imposing the concentration source constraints on the face and the edge respectively.

We relate material variation design in a material feature to a mass transport problem in a virtual diffusion process. Designers can utilize the intuition based on diffusion law and impose boundary conditions accordingly (in terms of material generation source, diffusivity, and material composition at particular spatial locations) as opposed to manipulate the material variation value at each control point in the native B-spline representation. We use these constraints to adjust material variation and preserve design intent. That is, the designer's role has been elevated from quantitatively manipulating control point to qualitatively imposing constraints. Our heterogeneous object modeling engine will then automatically compute the material variation value at each control point. Thus, many desirable B-spline properties such as local control are still preserved. If needed to be, users can still fine-tune the material variation through the B-spline representation. Since material variation and geometry are represented in a same B-spline solid, the design intent of material variation (constraints) is automatically maintained when the geometry is changed. Examples in Section 6 will further demonstrate this.

5. Constructive feature operations through direct face neighborhood alteration

Once synthesized material features are constructed, they need to be combined together to build the final part through three feature operations: addition, subtraction and partition. Given heterogeneous objects $A = \{A_1|A_2|\cdots|A_m\}$ and $B = \{B_1|B_2|\cdots|B_n\}$ and the feature operator $\otimes$, the resultant solid needs to be formed. It essentially includes two tasks:

- determine the boundary of A and B that appears in the resultant solid C (*Geometric Boundary Evaluation*), and
- organize the resultant faces into regions and associate material function m_i to each region g_i (*Material Region Forming*).

These feature operations can be transformed into traditional Boolean operations with appropriate processing of material variation attributes. However, in this research, both the geometric boundary evaluation and material region forming are conducted based on a novel method, direct face neighborhood alteration [20,24], to increase the modeling efficiency. We only briefly describe the concept here.

Neighborhood is a well-known concept from topology [1]. In heterogeneous objects, each face has two neighboring regions. We perceive a 3D face's neighborhood as a *two-sided face neighborhood* NF and represent it as a combination of two one-sided face neighborhood NF from each adjacent region.

Given the objects A and B, the faces from A and B, noted as F_A and F_B, can be classified against each other. There are five types of set membership classification (SMC) values: F_A in B, F_A out B, F_A on B/F_B on A, F_B in A, and F_B out A. Therefore, corresponding to the five SMC values, there are five NF operations for the operation $A\otimes B$: (1) $NF_A\otimes B_j$ for F_A inside region B_j, (2) $A\otimes NF_B$ for F_B inside region A_i, (3) $NF_A\otimes NF_B$ for F_A and F_B that are co-faces, (4) $NF_A\otimes B^C$ for F_A outside the object B, i.e. F_A interacts with region B^C, (5) $A^C\otimes NF_B$ for F_B outside the object A, i.e. F_B interacts with region A^C. Fig. 9 shows the five neighborhood operations. Since different regions have different material operation semantics, the NF operations are fulfilled by combining two separate one-sided NF operations, each of which operates according to the residing region's semantics.

F_A's neighborhood operation with region B_j can be represented as:

$$NF_A\otimes B_j = (nF_{A\mathrm{Front}}\otimes B_j)|(nF_{A\mathrm{Back}}\otimes B_j)$$

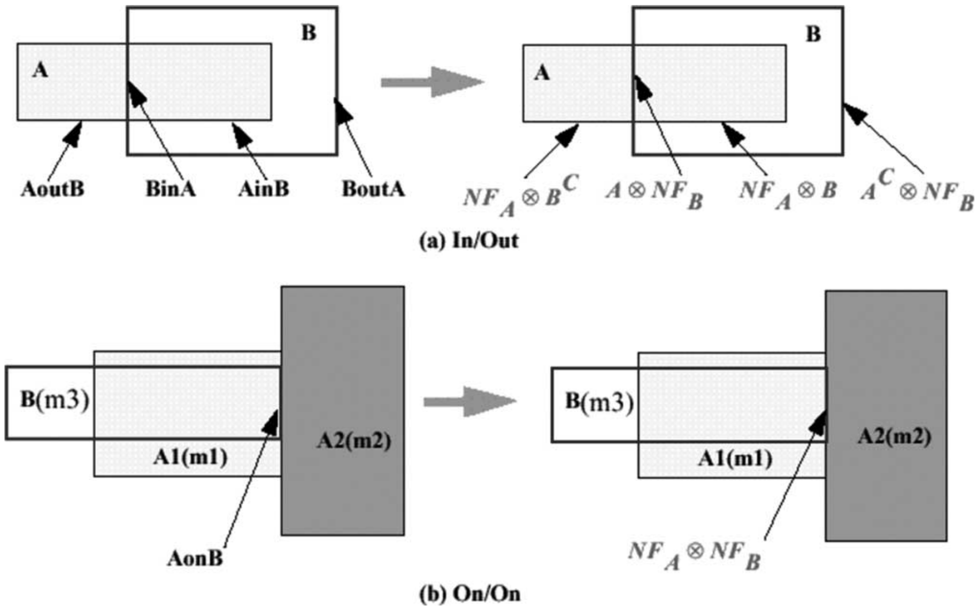

Fig. 9. Face membership classification and neighborhood operation.

Here $nF_{A\mathrm{Front}}$ and $nF_{A\mathrm{Back}}$ refer to the face F_A's front region and back region's neighborhood. For the generality, one-sided face neighborhood in region A_i is referred to as nF_{A_i}. The face neighborhood for the object A's complement set A^c is noted as nF_{A^c}.

An example of F_A interacting with region B is shown in Fig. 10 (bold line). From the four cases in the union operation, we have the following neighborhood alteration rules:

$$nF_{A_i} \cup B_j = \begin{cases} nF_{A_i} & mA = mB \\ nF_{A_i} & pA > pB \\ (\mathrm{dir}A_i, mB) & pA < pB \\ (\mathrm{dir}A_i, mA \oplus mB) & pA = pB \end{cases} \tag{11}$$

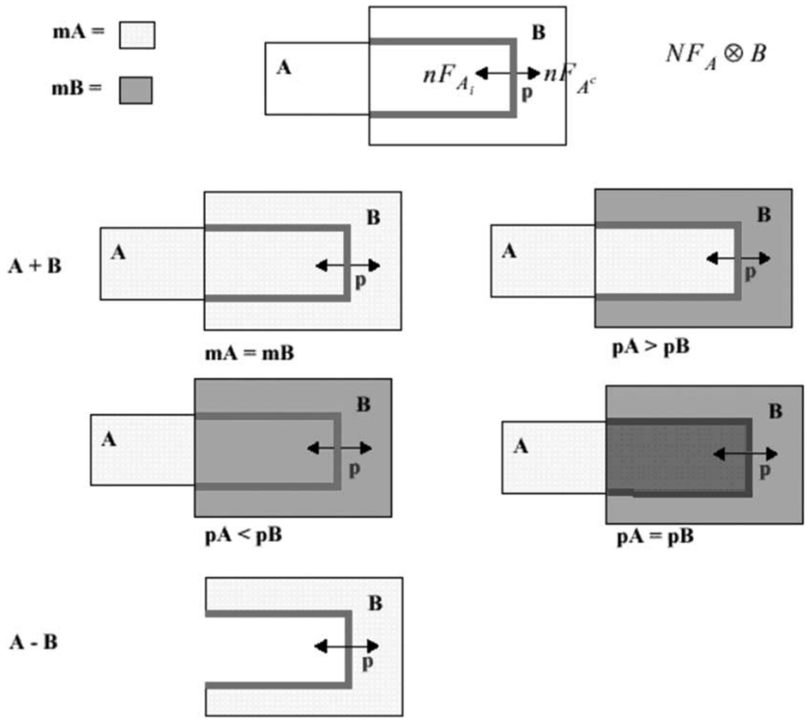

Fig. 10. Neighborhood operations for FA in B.

The other types of face neighborhood alteration can be derived similarly.

Both modeling tasks for constructive feature operations, boundary evaluation and material region forming, can then be derived from the altered neighborhood information.

6. Implementation and examples

A prototype system for feature based design methodology has been implemented using ACIS on a SUN Sparc workstation. This section presents the implementation result. Note some examples have appeared in our prior publications [23,24] and we describe in this paper how the feature based design method is used to design geometry and material variation within these parts.

6.1. Example 1: manipulating geometry while preserving design intent in a material feature

For each material primitive, the input of the system is a B-spline solid, consisting of a set of control points. The user interacts with system in two ways. First, the user can change system parameters, such as Q, the material source (material/unit volume) and D, the material diffusion coefficient. Second, the user can impose constraints. The two types of interaction processes continue until the user is satisfied with the result.

Fig. 11 shows an example of changing both geometry and material composition. The top of Fig. 11 is an initial B-spline solid, imposed with constraints on two boundary surfaces. Fig. 11a is the result. We can change the geometry of the solid and get the new solid in Fig. 11b and then impose composition constraints. This leads to a new solid in Fig. 11d. We can also impose the material composition constraints first (Fig. 11c) and then manipulate the geometry (Fig. 11d). This alteration of geometry and material composition manipulation sequence leading to the same result demonstrates the design intent can be preserved during the iterative design editing process.

6.2. Example 2: material properties directly conceivable to designers

A heterogeneous feature with graded materials, SiC and Al6061 alloy, is shown in Fig. 12. The thermal conductivities of the two materials are 180 and 25 W/mK. The strengths are $+145/-145$ MPa and $+0/-8300$ MPa. Using Eqs. (3) and (4), we can have the thermal conductivity and tensile/compression stress for each control point. These properties are respectively shown in Fig. 12. Fig. 12 also shows the values at the tip. Note, the notation $a/(b,c)$ in the figure means the value at the tip point is a while the minimal value of the whole volume is b, and the maximum value is c. The combination of SiC and Al alloy enables heat resistance and anti-oxidation properties on the high temperature side, mechanical toughness and strength on the low temperature

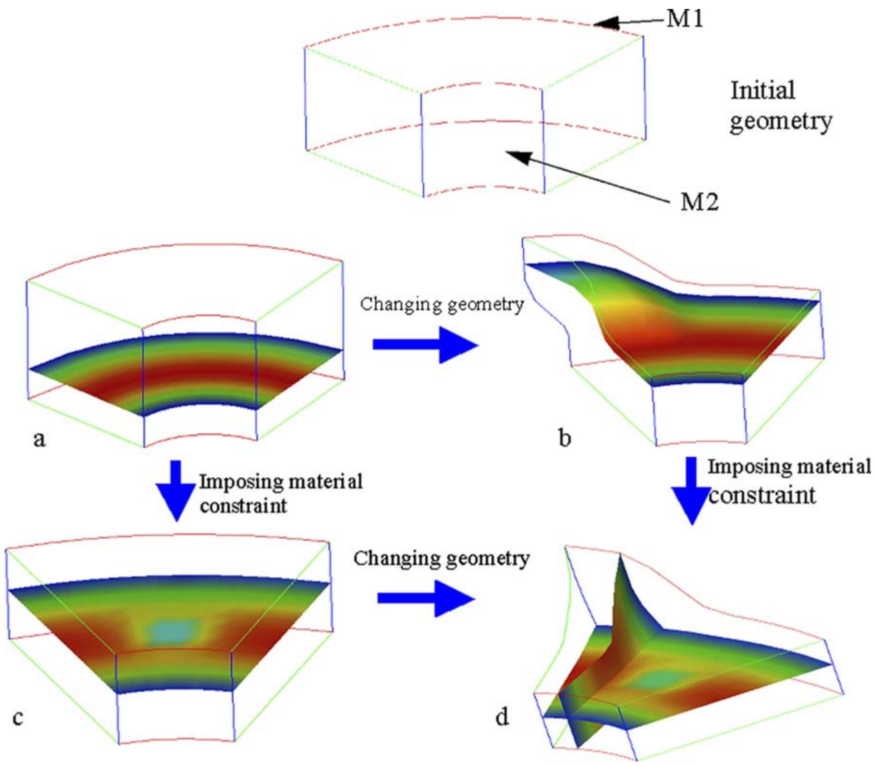

Fig. 11. Design intent is preserved during the iterative design process.

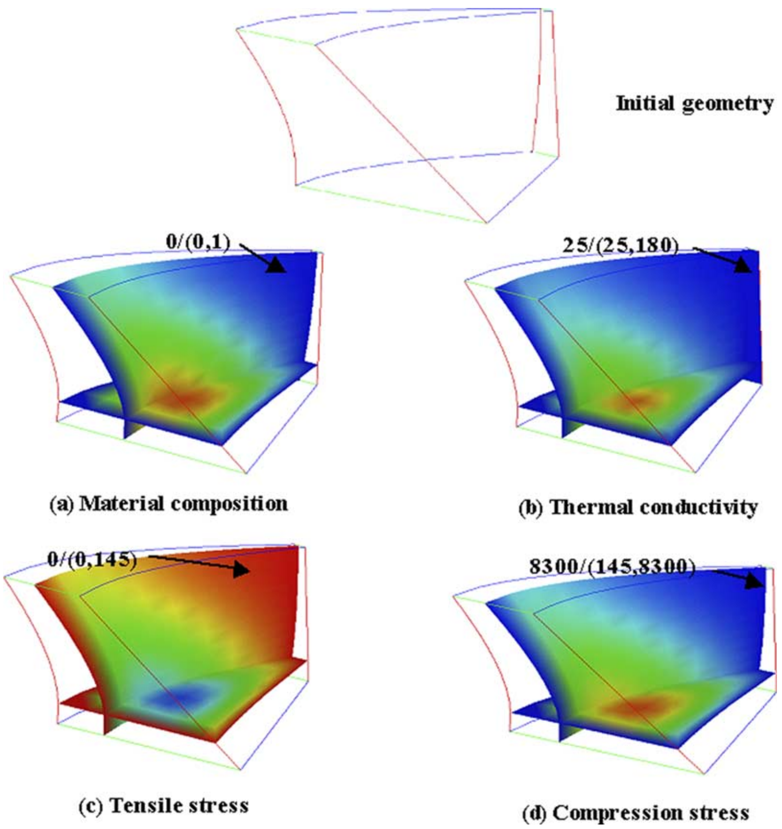

Fig. 12. Material properties directly conceivable to designers.

side, and effective thermal stress relaxation throughout the material.

Suppose the designer is not satisfied with the strength at the tip of turbine blade, the designer can choose to strengthen the tip by imposing constraints at the tip. The revised model is shown in Fig. 13, where the thermal conductivity has been changed from 25 to 140.94, tensile strength from 0 to 116.92 and compression strength from 8300 to 1724.37.

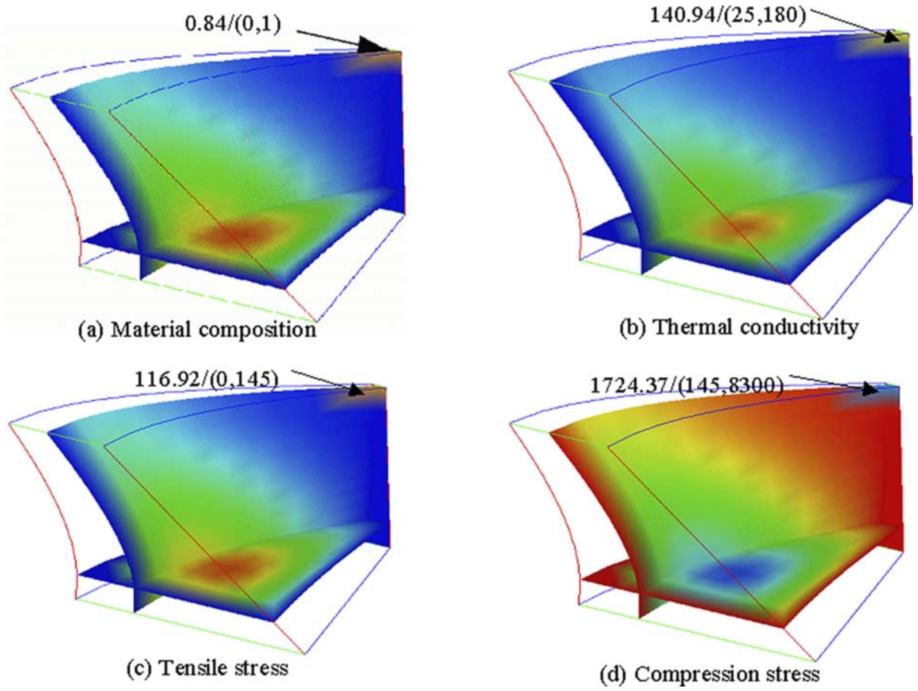

Fig. 13. Improving the tip strength at the tip.

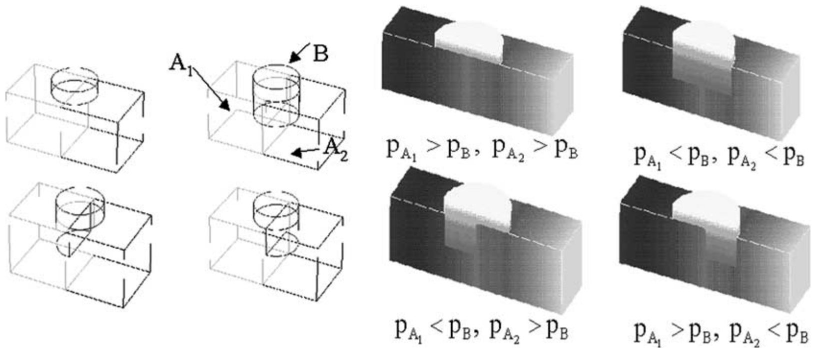

Fig. 14. Constructive feature operations.

Using this method, the designers directly interact with the system with familiar concepts (the material properties) rather than material composition. We believe this direct quantitative feedback of material properties is particularly useful for a designer during the design evolution process.

6.3. Example 3: constructive feature operations

Fig. 14 shows a sample part, consisting of two feature volumes, *A* and *B*. By direct face neighborhood alteration, the system gives different results, depending on the priority of each material feature primitive. The right half of the figure is the shaded cross-section of the parts. The feature operations are additive. Partition operations can also be used to get the same result.

6.4. Example 4: using feature based design method to simulate a MEMS fabrication process

Fig. 15 shows a MEMS fabrication process modeled through the system. Since MEMS fabrication involves geometric changes of different materials, a feature based design method can simulate the fabrication process. Two types of operations, additive and subtractive, are used. In this example, each fabrication step is represented in a material feature. The color changes illustrate the face neighborhood changes during the modeling process. In the last step (Fig. 15e) Electrode overrides Acetone. So all

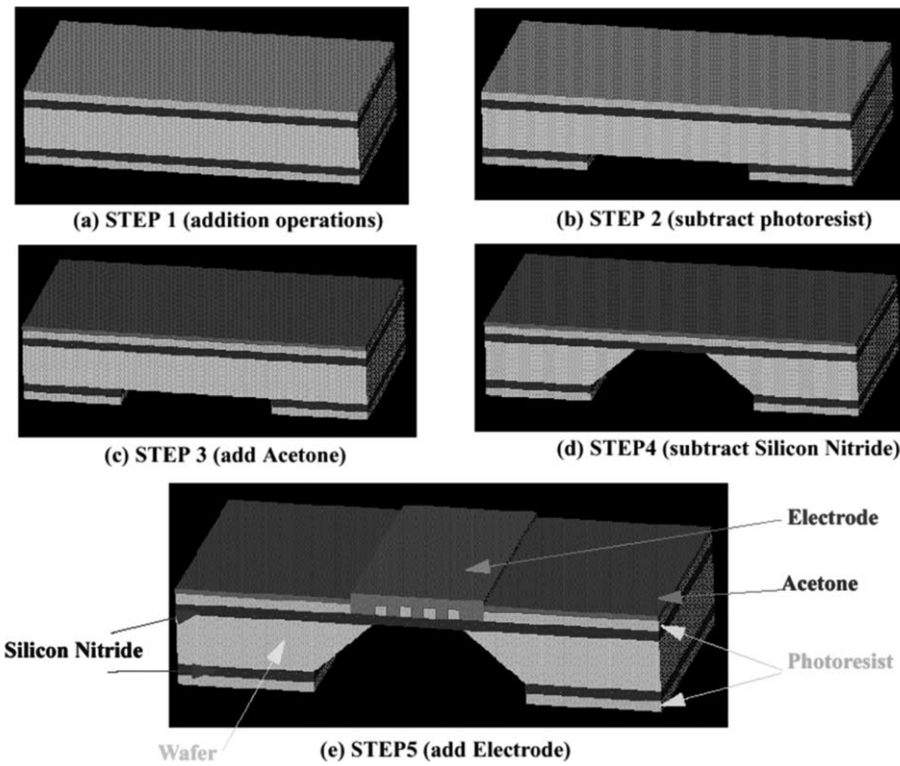

Fig. 15. Using FBD to simulate a MEMS fabrication process.

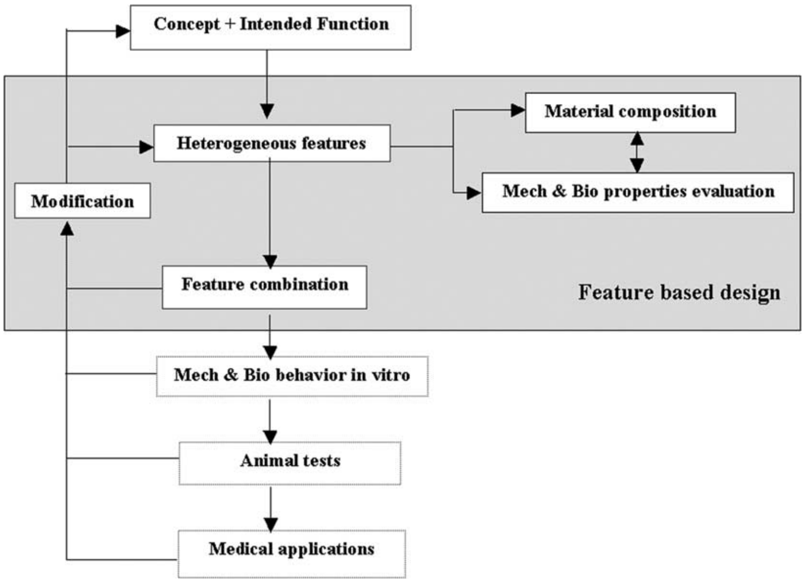

Fig. 16. Flowchart of a feature based design process for a new prosthesis.

the neighborhood of the faces from Acetone are changed to Electrode if they are 'inside' Electrode.

6.5. Example 5: using FBD to design a prosthesis structure

The following example of prosthesis design demonstrates a typical feature based design process for heterogeneous parts. Fig. 16 shows a flowchart for the prosthesis design process. Starting from the design functions, users select materials and form the heterogeneous material features, each of which is a B-spline volume. The feature combination algorithm combines these features into a heterogeneous object. After the mechanical and biological properties are obtained from the database for each individual material, these properties at each point in this prosthesis can then be evaluated. If users are not satisfied with the properties, they can select new material for each volume or change volume fractions. These steps of changing material composition of each feature in the constructive process form a feature based design process. After the property evaluation, property in vitro tests and animal tests are conducted before the designed prosthesis is used for medical purposes.

In the example of Fig. 17 is a prosthesis designed following the flow chart in Fig. 16. The materials are Titanium and graded HAp. Each of these design intents is represented as a separate B-spline volume (heterogeneous feature), such as in Region 2 and Region 7 in Fig. 17. In these two regions, pore and HAp are modeled as one material, while the Titanium is the other material. Because of the design intent of having a graded interface, we set the boundary conditions on inner surface and outer surface of the two regions. Regions 1 and 8 represent the bones. Regions 4 and 5 connect the two ends. Once the volume fraction for pore and HAp is known, another fraction is used to separate the pore and HAp. This fraction is constant throughout the region. The Fig. 17a and b show the graded porous structure and graded HAp, respectively, with

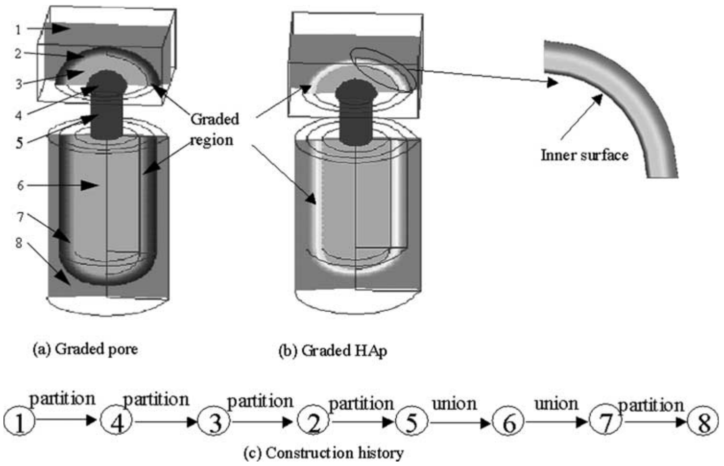

Fig. 17. Graded interface modeling within a prosthesis.

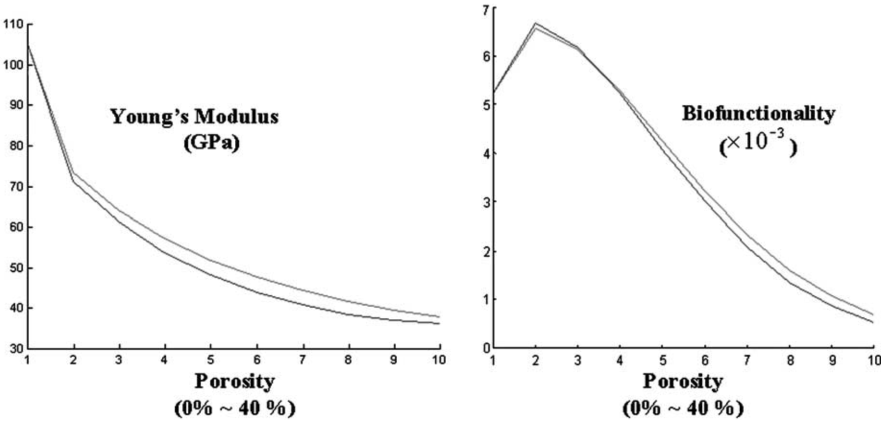

Fig. 18. Variation of Young's modulus and biofunctionality due to Q change.

the $M_{\text{pore}}/M_{HAp} = 0.5$. Fig. 17c shows the construction history. The partition in the construction history is similar to union operation but with the intersection region's material redefined. Modification to the material composition can lead to different Young's modulus and biofunctionality (BF) distribution throughout the region. In Fig. 18, we show the properties variation due to the change of Q (material generation source). These values are measured at different distance points from the inner surfaces of the graded regions.

This example demonstrates that the feature based design method not only provides an intuitive way to control the material compositions but also provides means to directly control the material properties. This draws a distinction from existing design methods for the prosthesis design, where material composition design and material property evaluation are conducted separately and sequentially.

7. Conclusions

This paper has addressed an important issue in heterogeneous object realization—feature based design methodology for heterogeneous.

In the context of heterogeneous object design, we propose the use of features to facilitate the high level (explicit) conceptualizing of geometric shape and material gradation. Based on our examination of the relationships between form features and material features in heterogeneous objects, a feature based design methodology is developed for heterogeneous object design. It is a constructive design process based on a set of user predefined heterogeneous features. The constructive feature operations include additive, subtractive and partition. To model material heterogeneity effectively and efficiently within each feature, a physics based B-spline heterogeneous object modeling method is researched and developed. In this method, B-spline representation is utilized to increase model coverage, and a physics process (diffusion process) is used to generate material composition profile to increase operation convenience. To speed up the efficiency of constructive feature operations, a direct face neighborhood alteration method is developed.

Our contributions in this paper include (1) the examination of the inherent relationships between form features and materials features, (2) the proposal of three synthesized features operations, and (3) the development of two enabling techniques for feature based design: physics-based B-spline object heterogeneity modeling and direct face neighborhood alteration for constructive feature operations.

Acknowledgements

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