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Design of heterogeneous turbine blade

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Abstract

Constantly rising operating pressure and temperature in turbine drivers push the material capabilities of turbine blades to the limit. The recent development of heterogeneous objects by layered manufacturing offers new potentials for the turbine blades. In heterogeneous turbine blades, multiple materials can be synthesized to provide better properties than any single material. A critical task of such synthesis in turbine blade design is an effective design method that allows a designer to design geometry and material composition simultaneously.

This paper presents a new approach for turbine blade design, which ties B-spline representation of a turbine blade to a physics (diffusion) process. In this approach, designers can control both geometry and material composition. Meanwhile, material properties are directly conceivable to the designers during the design process. The designer's role is enhanced from merely interpreting the optimization result to explicitly controlling both material composition and geometry according to the acquired experience (material property constraints).

The mathematical formulation of the approach includes three steps: using B-spline to represent the turbine blade, using diffusion equation to generate material composition variation, using finite element method to solve the constrained diffusion equation. The implementation and examples are presented to validate the effectiveness of this approach for heterogeneous turbine blade design.

Keywords: Heterogeneous object design; B-spline; Turbine blade; Physics based modeling

1. Introduction

Constantly rising operating pressure and temperature in turbine drivers push the material capabilities of turbine blades to the limit. Turbine blades, particularly those used in aircraft engines, are subject to severe thermo-mechanical loading, which leads to intense thermal stresses. Initial designs of these components used metals on low temperature side and ceramics on high temperature side. However, the property difference between the two materials generated high stress concentrations at the interfaces, which resulted in cracks, plastic deformations and interfacial de-cohesion.

To ideally resolve these issues, the turbine blade materials must possess the following properties—heat resistance and anti-oxidation properties on the high temperature side, mechanical toughness and strength on the low temperature side, and effective thermal stress relaxation throughout the material.

The recent advances in layered manufacturing and the concept of functionally gradient materials offer just these material properties. With these advances, now it is possible to fabricate a new type of turbine blade, heterogeneous turbine blade, to resolve the issue of the material property difference. A heterogeneous turbine blade is made of different constituent materials and possesses gradient material properties. The layered manufacturing techniques refer to a host of fabrication processes that build parts by depositing material layer-by-layer. These processes are now capable of fabricating truly three-dimensional (3D) heterogeneous objects where material variations are three-dimensional. In these heterogeneous objects, different material properties from different constituent materials can be synthesized and exploited to enhance performance.

An example of such a heterogeneous turbine blade design is shown in Fig. 1 [11]. The sharp interface between the metal and ceramic is eliminated by using a graded zone of metal/ceramic, denoted as FGM in Fig. 1. In such a structure, the properties can be adjusted by controlling the composition, microstructure and porosity ratios from metal to ceramic. The graphs in the figure show typical variation in properties due to the variation in material composition at the FGM region.

To obtain a heterogeneous turbine blade model for analysis and fabrication, an efficient design method is needed. Existing turbine blade design methods are not suitable for heterogeneous turbine blade design. They

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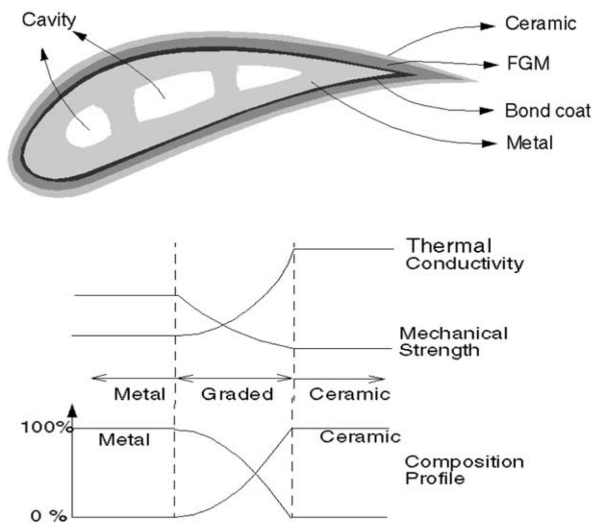

Fig. 1. Turbine blade and the material properties [11].

typically employ an optimizer in combination with a finite element analysis package. Such an optimization approach is computationally expensive. In general, such an optimization approach limits the role of the designer in the design process. With the introduction of the added dimension—material variation—to the design space of heterogeneous turbine blade, the computation cost is exacerbated. One of the main challenges lies in the fact there exist a large number of degrees of freedom to completely define a heterogeneous object. For example, for a 3D object with m types of materials and n number of variations for each material, the modeling space is $E^3 \times n^m$ dimensional. That is to say, for a turbine blade made of modestly three constituent materials with volume fraction 2% as the resolution, there are 1.25×10^5 possible designs even with geometry assumed to be the same. Due to such heavy computation involved, this approach becomes practically very challenging even with the advanced computational power.

To resolve this issue, this paper proposes a different approach for heterogeneous turbine blade design. In this approach, the design process is guided by the experienced designers to save expensive computations. The role of the designer is enhanced from merely interpreting the results of an automated system to actively guiding the design of the geometry and material composition variation within the turbine blade. The designers change the material variation according to a simulated physics (diffusion) process. They can intuitively control the material composition with only a few parameters that carry physical meanings, such as a diffusion coefficient.

The remainder of the paper is organized as follows. Section 2 reviews the previous research pertaining to turbine blade design and heterogeneous object design in general. Section 3 proposes the overall design process for heterogeneous turbine blade. Section 4 presents the B-spline representation for turbine blade material composition and the corresponding material properties. Section 5 details the mathematical formulation for the physics (diffusion) based heterogeneous object modeling (HOM) method. Implementation and some examples of turbine blade modeled through the physics based HOM engine are presented in Section 6. Finally, this paper is concluded in Section 7.

2. Literature review

The design of a high-efficiency turbomachinery blade is a complex task. Due to its heavy computation in 3D blade design/analysis calculation, it requires an engineering effort that is quite significant for a small company. Consequently, a two-dimensional (2D) cascade simulation is often used before embarking on a full-dimensional analysis. A genetic optimizer that modifies 2D blade profile and calculates its profile loss is developed in Ref. [19]. A knowledge-based method using an artificial neural network is developed to construct an approximate model for all previous designs of the blade [16]. Finite element analysis program is used to optimize a composite wind turbine blade [2].

The need to further improve the machinery performance requires the use of 3D Navier–Stokes solvers during the design process. These solvers, however, do not indicate what geometry modifications are required to improve the blade performance. The search for optimized blades must therefore be guided either by an experienced designer or by a numerical method. This often requires a large number of Navier–Stokes computations, to evaluate many different blade geometries, before reaching a good solution satisfying both the aerodynamic and the mechanical requirements. Although this procedure allows the design of very efficient blades, it is expensive in terms of computation and/or operator time. To reduce the number of Navier–Stokes computations, an Artificial Neutral Network is used to reduce the computational effort [16].

All these methods do not deal with the material composition variation within the turbine blade. Directly applying these methods for both geometry and material composition design can only exacerbate the computational cost.

Research on heterogeneous object design has been primarily focusing on the representation and modeling scheme. Kumar and Dutta proposed R-m sets for representing heterogeneous objects [11]. Jackson et al. proposed another modeling approach based on subdividing the solid model into sub-regions and associating the analytical composition blending function with each region [9]. Some other modeling and representation schemes, such as utilizing voxel model, implicit functions and texturing, have also been proposed [13,21].

Design methodologies for heterogeneous objects are just beginning to emerge [17]. Typically, a numerical analysis

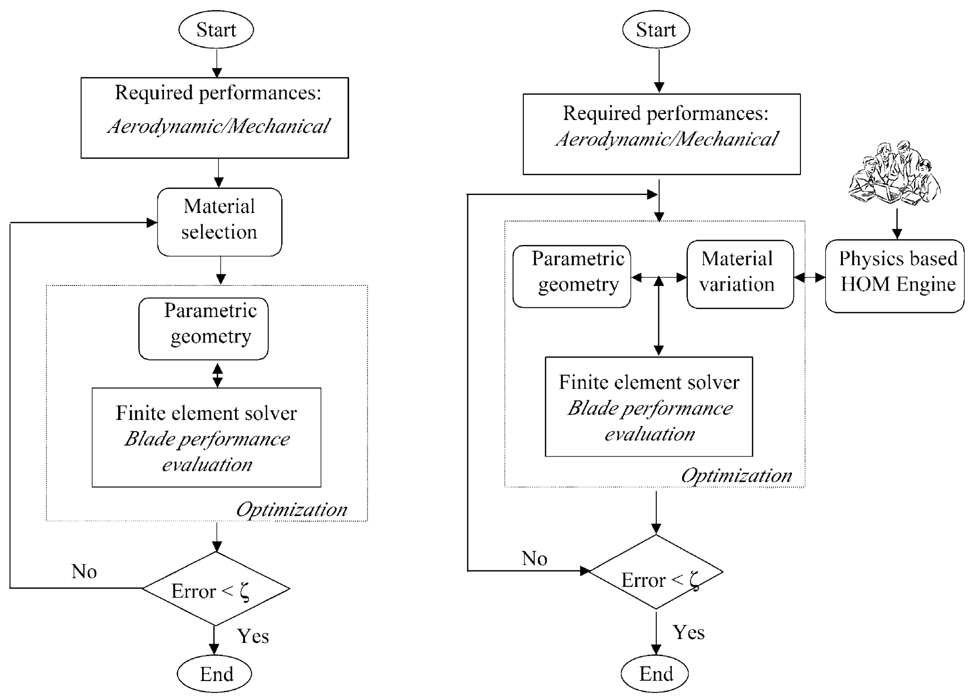

Fig. 2. Design process comparison, (a) Conventional design process, (b) Design process for heterogeneous turbine blade.

based multi-objective optimization method is used to design a heterogeneous component. For example, an approach for heterogeneous flywheel design based on optimization is reported in Ref. [8].

The early methods for the fabrication of heterogeneous objects include powder metallurgy, physical and chemical vapor deposition, plasma spraying, self-propagating high temperature synthesis (SHS) and galvanoforming. Recently, several fabrication methods have been developed that are capable of manufacturing heterogeneous objects in which the material variation is 3D. For example, recently abrasive turbine blade tips have been fabricated by direct laser fabrication with microstructure, properties and performance superior to conventional production material [4]. The ultimate goal of these processes would enable the direct fabrication of functionally graded microstructures with properties tailored to specific regions of the component. These new fabrication methods can be broadly referred to under the term 'layered manufacturing' (LM). A host of LM technologies are currently available commercially. A non-exhaustive list includes: stereolithography (SLA) by 3D Systems, selective laser sintering (SLS) by DTM Corp., fused deposition modeling (FDM) by Stratasys Corp., solid ground curing (SGC) by Cubital, and laminated object manufacturing (LOM) by Helisys. In addition, several LM processes are under development at various universities, such as Carnegie Mellon, Stanford, MIT, University of Dayton, University of Michigan, and the University of Texas. Refer to Dutta et al. [5] for details of these LM processes.

3. Overall design process for heterogeneous turbine blade

Conventionally, designers begin with required aerodynamic and mechanical performance (Fig. 2a). They select materials and initial turbine blade geometry. The geometry is typically Bézier or B-spline geometry. For each set of blade geometry, a 2D or 3D finite element analysis is conducted to evaluate the blade performance. To obtain an optimal performance, an optimization program, such as genetic algorithm or simulated annealing, is often used to obtain the blade geometry.

With the introduction of heterogeneous turbine blade, the volume fractions of the constituent materials have also become the design variables along with the constituent material type. Naïve designers may still use an optimization program to find out the best blade geometry and material composition. However, due to the formidable size of degrees of freedom to specify the material composition, an optimal solution may not be obtained cost effectively. This paper presents a new approach (Fig. 2b), in which the material composition can be designed by an experienced designer through a physics based HOM method. It allows designers to intuitively control the material variation with only a few parameters that carry engineering meanings. Meanwhile, the material properties of the turbine blade are conceivable to designers while they change the material variations.

Fig. 3 shows the flowchart of physics-based B-spline HOM process. The input of the system is a B-spline solid, consisting of a set of control points. The user interacts with

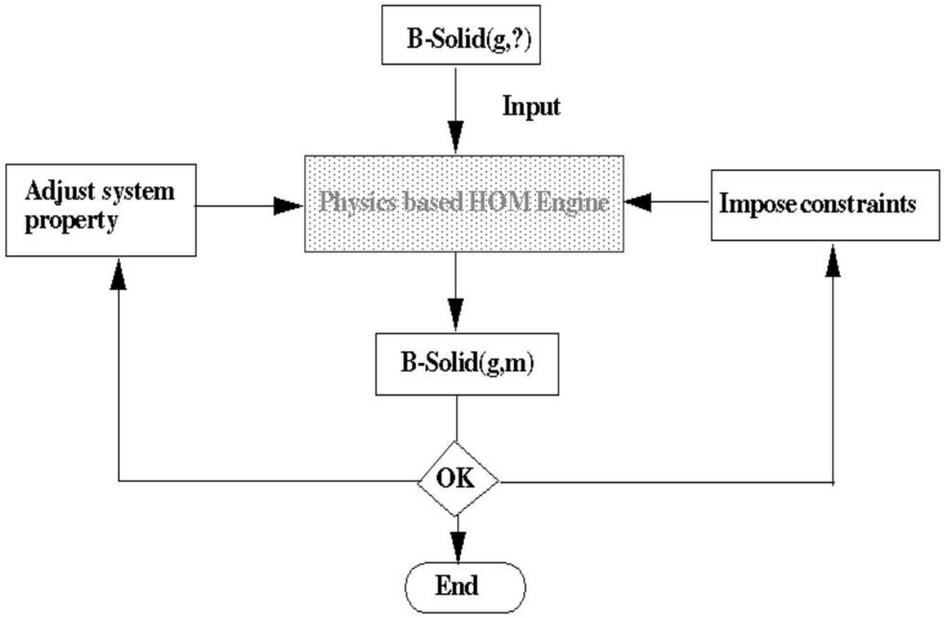

Fig. 3. Flowchart of physics based HOM.

system in two ways. First, the user can change system parameters, such as Q, the material source (material/unit volume) and D, the material diffusion coefficient. Second, the user can impose constraints. The two types of interaction processes continue until the user is satisfied with the result.

4. B-Spline representation for heterogeneous objects

Tensor product solid representation has been widely used in computer aided geometry design community. Under the context of heterogeneous objects, relevant proposals based on tensor product volumes have also been reported [9,21].

The curves and surfaces of turbine blade in general have a Bézier or a B-spline representation [7,16]. In this paper, B-spline tensor representation is used to represent material composition and material properties in heterogeneous turbine blades. We choose B-spline as a representation scheme for heterogeneous objects simply to shorten the computational time. There is no technical difficulty to extend the methodology in this paper to NURBS volumes.

4.1. B-spline tensor solid representation for heterogeneous objects

For each point (u, v, w) in the parametric domain of a tensor product B-spline volume V (Fig. 4), there is a corresponding point $V(u, v, w)$ at Cartesian coordinates (x, y, z) with material composition M, noted as (x, y, z, M). We define

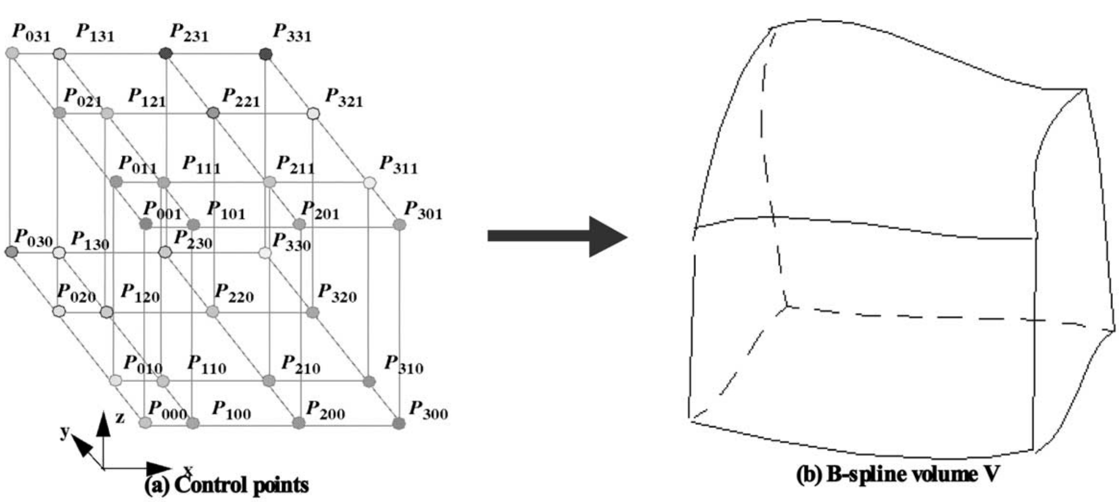

Fig. 4. Tensor product B-spline volume.

such a B-spline volume as:

$$V(u,v,w) = \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} N_{i,p}(u)N_{j,q}(v)N_{k,r}(w)P_{i,j,k} \quad (1)$$

where $P_{i,j,k} = (x_{i,j,k}, y_{i,j,k}, z_{i,j,k}, M_{i,j,k})$ are control points for the heterogeneous solid volume. $N_{i,p}$, $N_{j,q}$ and $N_{k,r}$ are the pth-degree, qth-degree and rth-degree B-spline functions defined in the direction of u, v, w, respectively. For example, $N_{i,p}(u)$, the ith B-spline basis function of p-degree (order $p+1$), is defined as:

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p-1} - u_i} N_{i,p-1}(u) + \frac{u_{i+p} - u}{u_{i+p} - u_{i+1}} N_{i+1,p-1}(u)$$

4.2. Representation for material properties of heterogeneous objects

Due to material gradation in heterogeneous objects, material properties also exhibit variation. For designers, controlling an heterogeneous object's properties is more useful and intuitive than controlling the material composition. In order to control material properties, we need an effective representation scheme.

The relationship between material properties and composition has been extensively studied [12]. For example, Eqs. (2) and (3) give the approximate relationships of thermal conductivities and mechanical strengths versus material composition. M_a, M_b are volume fractions of two composite materials at each point. λ_a, λ_b are the thermal conductivities and S_a, S_b are strengths for two materials a and b.

$$\lambda = \lambda_a M_a + \lambda_b M_b + M_a M_b \frac{\lambda_a - \lambda_b}{3/(\lambda_b/\lambda_a - 1) + M_a} \quad (2)$$

$$S = S_a \cdot M_a + S_b \cdot M_b \quad (3)$$

Such property variations can be tailored to achieve powerful functions under various loading conditions as evidenced in many functionally gradient materials. We generalize the relationship between material property and material composition as follows:

$$E = f(E_a, E_b, M_a, M_b) \quad (4)$$

where E is the material property of interest, E_a and E_b are material properties of material a and b and M_a, M_b are volume fractions.

Combining the material property equation and Eq. (1), we can have the B-spline representation for material properties:

$$E(u,v,w) = \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} N_{i,p}(u)N_{j,q}(v)N_{k,r}(w)E_{i,j,k} \quad (5)$$

where $E_{i,j,k}$ is material property at each control point. It can be obtained from Eq. (4).

In this paper, we assume the same set of geometric control points can be used for both material composition model and property model. For a complex heterogeneous object, a single approximation function may not hold. Several sub-functions are used to describe the relationship between property and volume fraction. However, with the existing rich algorithms for control points and knots inserting for B-spline representation, the same set of control points can still be used to represent both material composition and property variation. Therefore, the assumption still holds.

5. Diffusion-based heterogeneous object modeling

5.1. Mathematical model for diffusion process

With B-spline as a representation for heterogeneous objects, we can now proceed to the investigation of material composition profile generation. In this section, we describe how diffusion process generates different material composition profile. Diffusion is a common physical process for the formation of material heterogeneity:

- many coating processes for turbine blade, such as doping and vapor deposition, can be characterized as diffusion process [18].
- in integrated circuit fabrication, diffusion has been the primary method for introducing impurities such as boron, phosphorus, and antimony into silicon to control the majority-carrier type and resistivity of layers formed in the wafer [10].
- in the drug delivery from a polymer, the drug release can be described in most cases by diffusion [14].

In all these diffusion processes, the controlled volume ratios (or particle concentration in some context) throughout the problem domain are controlled to certain profiles to achieve the respective objectives. This is the rationale we chose the diffusion process as the underlying physical process for HOM. We can use diffusion process to intuitively control the volume rations in heterogeneous objects.

The mathematical modeling of controlled material composition in these processes is based on the Fick's laws of diffusion.

Fick's first law of diffusion states that the particle flow per unit area, q (called particle flux), is directly proportional to the concentration gradient of the particle:

$$q = -D \cdot \frac{\partial M(x,t)}{\partial x} \quad (6)$$

where D is the diffusion coefficient and M is the material composition (particle concentration).

Fick's second law of diffusion can be derived using the continuity equation for the particle flux: $\partial M/\partial t = -\partial q/\partial x$.

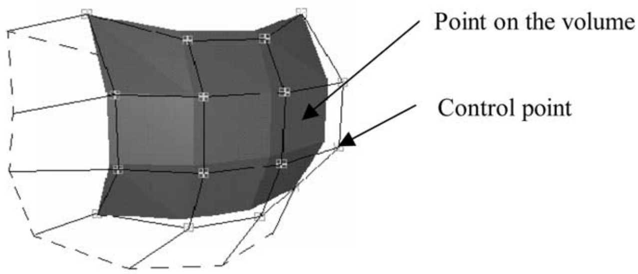

Fig. 5. B-spline control point and point on the B-spline volume.

That is, the rate of increase of concentration with time is equal to the negative of the divergence of the particle flux. Combining these equations, we have Fick's second law:

$$\frac{\partial M}{\partial t} = D\cdot\frac{\partial^2 M}{\partial x^2} \tag{7}$$

For a given volume, the amount of material is conserved during the diffusion process. In other words, the rate of change of the material M within the volume must equal to the local production of M within the volume plus the flux of the material across the boundary. Mathematically,

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega M\,\mathrm{d}\Omega = \int_\Omega Q\,\mathrm{d}\Omega - \int_S q_i n_i\,\mathrm{d}S \tag{8}$$

where Q is the material generated per unit volume.

Applying Fick's laws into the earlier equation and using the divergence theorem, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega M\,\mathrm{d}\Omega = \int_\Omega Q\,\mathrm{d}\Omega - \int_S \frac{\partial q_i}{\partial x_j}\mathrm{d}S.$$

After dropping the integral over volume, we have

$$\frac{\mathrm{d}M}{\mathrm{d}t} = Q + \frac{\partial}{\partial x_i}\left(D_{ij}\cdot\frac{\partial M}{\partial x_j}\right) \tag{9}$$

5.2. Finite element approximation for steady state equation

In this section, we combine B-spline representation and diffusion equation and derive the equations for diffusion based B-spline HOM.

To model a heterogeneous object using diffusion process, we concentrate on the steady state of the diffusion process (a static volume). We then extend it to the time dependent objects.

In a static volume, there is no material change over time, i.e. $\mathrm{d}M/\mathrm{d}t = 0$. Hence Eq. (9) becomes

$$Q + \frac{\partial}{\partial x_i}\left(D_{ij}\cdot\frac{\partial M}{\partial x_j}\right) = 0 \tag{10}$$

which has essentially the same format as many other steady-state field problems governed by the general 'quasi-harmonic' equation, the particular cases of which are the well-known Laplace and Poisson equations. The range of physical problems falling into this category is large. To name a few, there are heat conduction and convection, seepage through porous media, irrotational flow of ideal fluid, distribution of electrical or magnetic potential, torsion of prismatic shaft, etc. [3].

If we abbreviate Eq. (1) as $V(u,v,w) = N\cdot P$, and we substitute the material composition M component from Eq. (1) into Eq. (10), we have the diffusion equation for the B-spline model:

$$Q + \frac{\partial}{\partial x_i}\left(D_{ij}\cdot\frac{\partial (N\cdot P)}{\partial x_j}\right) = 0 \tag{11}$$

For any given B-spline volume V, there are $(n+1)\times(m+1)\times(l+1)$ control points. That is, there are $(n+1)\times(m+1)\times(l+1)$ degrees of freedom for Eq. (11). In Eq. (11), we assume the positions (x,y,z) of control points for the solid V are given. Our objective is to calculate the material composition M at each point.

Solving Eq. (11) leads to the solution to material composition for a heterogeneous object. Direct analytical solution for such second order differential equations is difficult to obtain. Instead, we use finite element technique to solve the earlier equations. What differs this method from most standard FE methods are: (1) the unknown are control points rather than points on the volume. As shown in Fig. 5, the points on the B-spline volume and the control points are different, (2) the shape functions are B-spline rather than typical FEM shape functions. The following is a brief presentation of finite element approximation for diffusion based B-Spline HOM. Refer to Cheung et al. [3] and Heinrich et al. [6] for the specific details of FE formulation.

5.2.1. Weak form of quasi-form equation

We present Eq. (11) in the following form for finite element approximation:

Governing equation:

$$Q + \frac{\partial}{\partial x_i}\left(D_{ij}\cdot\frac{\partial M}{\partial x_j}\right) = 0 \tag{12}$$

(i) forced boundary condition:

$$\phi = \phi_0 \text{ on } \Gamma_1 \tag{13}$$

(ii) natural boundary condition:

$$q_n = \vec{q}\cdot\hat{n} \text{ on } \Gamma_2 \tag{14}$$

To create the finite element approximation for Eq. (12), we need to convert the partial differential equations into its weak form. The second order differential equation can be turned into its weak form for an arbitrary function Ψ by integrating it partly.

$$\int_\Omega D_{ij}(\nabla\Psi)\cdot\frac{\partial M}{\partial x_j}\mathrm{d}\Omega = -\int_{\Gamma_2}\Psi q_n\,\mathrm{d}\Gamma + \int_\Omega \Psi Q\,\mathrm{d}\Omega \tag{15}$$

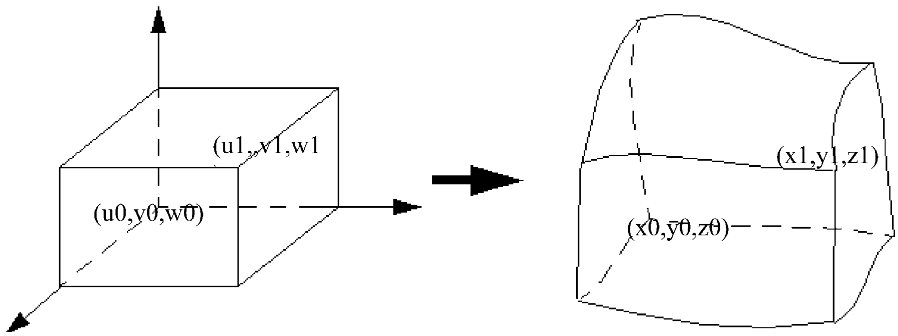

Fig. 6. Element in B-spline solid.

5.2.2. Galerkin approximation

By the Galerkin approximation, the weighting function Ψ is defined with the aid of the element interpolation function (B-spline function) N_α such that $\Psi = N_\alpha \Psi_\alpha$, where Ψ_α is the value of the function Ψ at the control node α. So we have

$$\int_\Omega N_{\alpha,i} D_{ij} \frac{\partial M}{\partial x_j} \mathrm{d}\Omega = \int_\Omega N_\alpha Q \, \mathrm{d}\Omega - \int_{\Gamma_2} N_\alpha q_n \, \mathrm{d}\Gamma \tag{16}$$

5.2.3. B-Spline heterogeneous volume discretization

We represent the B-spline heterogeneous solid as a set of elements where each element is associated with a set of degrees of freedom. The element boundaries are defined by the known locations within the underlying B-spline basis functions and the degrees of freedom are the B-spline's geometry and material control points. For example, in Fig. 6, a cubic in parametric domain corresponds to a 3D volume in the physical coordinate system.

Let us note

$$M(u,v,w) = \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=0}^{l} N_{i,p}(u) N_{j,q}(v) N_{k,r}(w) M_{i,j,k} \tag{17}$$

as $M(u,v,w) = NM_{i,j,k}$, here we let N be the shape function, M be the material composition control points.

By the finite element approximation, Eq. (16) becomes

$$\left[\int_\Omega \frac{\partial N_\alpha^m}{\partial x_i} \cdot D_{ij} \frac{\partial N_\beta^m}{\partial x_j} \right] M = \int_\Omega N_\alpha^m Q \, \mathrm{d}\Omega - \int_{\Gamma_2} N_\alpha^m q_n \, \mathrm{d}\Gamma \tag{18}$$

in matrix form, Eq.(18) becomes

$$KM = \vec{B} - \vec{S} \tag{19}$$

where

$$\underset{N\times N}{K_\mathrm{e}} = \left[k \int_{\Omega_\mathrm{e}} \left(\frac{\partial N_i}{\partial x} \cdot \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \cdot \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \cdot \frac{\partial N_j}{\partial z} \right) \mathrm{d}\Omega \right] \tag{20}$$

$$\underset{N\times 1}{\vec{B}_e} = \left[\int_{V_\mathrm{e}} N^m Q \, \mathrm{d}V \right], \qquad \underset{N\times 1}{\vec{S}_e} = \left[\int_{\Gamma_\mathrm{e}} N^m q_n \, \mathrm{d}\Gamma \right] \tag{21}$$

With function Q and q interpolated in terms of the nodal values, we have

$$\underset{N\times 1}{\vec{B}_e} = \left[\int_{\Omega_\mathrm{e}} N_i^m N_j^m \, \mathrm{d}\Omega \right] Q_j,$$

and

$$\underset{N\times 1}{\vec{S}_e} = \left[\int_{\Gamma_\mathrm{e}} N_i^m N_j^m \, \mathrm{d}\Gamma \right] q_j$$

K_e is the element stiffness matrix, and $\vec{B}_\mathrm{e}$ is the element body force and $\vec{S}_\mathrm{e}$ is the element surface force.

It is not difficult to prove that the element stiffness matrix is symmetric and positive definite. Solving Eq. (19) will give the unknown values $M_{i,j,k}$—the material composition control points in the solid.

5.2.4. Transformation of differential operator

To calculate the partial differential equation, e.g. $\partial N_i / \partial x$ in the matrix K, we need to use differential property of B-spline basis function and the chain rule of partial differential.

Let

$$P = \sum_{i=0}^{n} \sum_{j=0}^{m} \sum_{k=0}^{l} N_{i,p}(u) N_{j,q}(v) N_{k,r}(w) P_{i,j,k},$$

from B-spline basis function property [15], we know

$$\begin{cases} \dfrac{\partial N}{\partial u} = \displaystyle\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} N'_{i,p}(u) N_{j,q}(v) N_{k,r}(w) \\ \dfrac{\partial N}{\partial v} = \displaystyle\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} N_{i,p}(u) N'_{j,q}(v) N_{k,r}(w) \\ \dfrac{\partial N}{\partial w} = \displaystyle\sum_{i=1}^{n} \sum_{j=1}^{m} \sum_{k=1}^{l} N_{i,p}(u) N_{j,q}(v) N'_{k,r}(w) \end{cases}$$

according to the chain rule of partial differential equation, we can write

$$\begin{cases} \dfrac{\partial N}{\partial u} = \dfrac{\partial N}{\partial x}\cdot\dfrac{\partial x}{\partial u} + \dfrac{\partial N}{\partial y}\cdot\dfrac{\partial y}{\partial u} + \dfrac{\partial N}{\partial z}\cdot\dfrac{\partial z}{\partial u} \\ \dfrac{\partial N}{\partial v} = \dfrac{\partial N}{\partial x}\cdot\dfrac{\partial x}{\partial v} + \dfrac{\partial N}{\partial y}\cdot\dfrac{\partial y}{\partial v} + \dfrac{\partial N}{\partial z}\cdot\dfrac{\partial z}{\partial v} \\ \dfrac{\partial N}{\partial w} = \dfrac{\partial N}{\partial x}\cdot\dfrac{\partial x}{\partial w} + \dfrac{\partial N}{\partial y}\cdot\dfrac{\partial y}{\partial w} + \dfrac{\partial N}{\partial z}\cdot\dfrac{\partial z}{\partial w} \end{cases}$$

If we note Jacobian matrix as

$$J = \frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{bmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial y}{\partial u} & \dfrac{\partial z}{\partial u} \\ \dfrac{\partial x}{\partial v} & \dfrac{\partial y}{\partial v} & \dfrac{\partial z}{\partial v} \\ \dfrac{\partial x}{\partial w} & \dfrac{\partial y}{\partial w} & \dfrac{\partial z}{\partial w} \end{bmatrix},$$

we have

$$\begin{bmatrix} \dfrac{\partial N}{\partial u} \\ \dfrac{\partial N}{\partial v} \\ \dfrac{\partial N}{\partial w} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial y}{\partial u} & \dfrac{\partial z}{\partial u} \\ \dfrac{\partial x}{\partial v} & \dfrac{\partial y}{\partial v} & \dfrac{\partial z}{\partial v} \\ \dfrac{\partial x}{\partial w} & \dfrac{\partial y}{\partial w} & \dfrac{\partial z}{\partial w} \end{bmatrix} \cdot \begin{bmatrix} \dfrac{\partial N}{\partial x} \\ \dfrac{\partial N}{\partial y} \\ \dfrac{\partial N}{\partial z} \end{bmatrix} = J\cdot \begin{bmatrix} \dfrac{\partial N}{\partial x} \\ \dfrac{\partial N}{\partial y} \\ \dfrac{\partial N}{\partial z} \end{bmatrix} \tag{22}$$

Hence

$$\frac{\partial N}{\partial x}, \frac{\partial N}{\partial y}, \frac{\partial N}{\partial z}$$

can be achieved from the following:

$$\begin{bmatrix} \dfrac{\partial N}{\partial x} \\ \dfrac{\partial N}{\partial y} \\ \dfrac{\partial N}{\partial z} \end{bmatrix} = J^{-1}\cdot \begin{bmatrix} \dfrac{\partial N}{\partial u} \\ \dfrac{\partial N}{\partial v} \\ \dfrac{\partial N}{\partial w} \end{bmatrix} \tag{23}$$

5.3. Heterogeneous object modeling by imposing constraints

The earlier formulation has provided a methodology to calculate the material composition for diffusion process. It can be generalized to manipulate material composition of B-spline heterogeneous solid objects by imposing constraints.

The constraints that are imposed on the B-solid include the heterogeneity information on the boundary, or the heterogeneity at specific location, or any other type of constraints that can be transformed into a set of equations.

For the constraints imposed on the geometry boundary, e.g. $M_{u=0} = M_0$. This type of constraints can be directly mapped to the heterogeneity of control points.

The second type of constraints can be converted to a set of linear equations. For example, $M(u_i, v_i, w_i) = M_i$, which can be represented as a linear equation of variable $M_{i,j,k}$ from Eq. (17).

Here, we consider these constraints as a set of linear equations:

$$A\cdot M = E \tag{24}$$

The matrix equation in Eq. (19) can be mathematically transformed into a linearly constrained quadratic optimization problem:

$$\min_p \left\| \frac{1}{2} M^T K M - M^T B \right\| \tag{25}$$

where p is the control point set for the B-spline solid.

To accommodate the constraints in Eq. (24), solution methods generally transform this to an unconstrained system [20]: $\min\|(1/2)M^{\Delta T}K^{\Delta}M^{\Delta} - M^{\Delta T}B^{\Delta}\|$, whose solutions M^{Δ}, when transformed back to M, are guaranteed to satisfy the constraints. The unconstrained system is at a minimum when its derivatives are 0, thus we are led to solve the system $K^{\Delta}M^{\Delta} = B^{\Delta}$.

Specifically, we introduce a Lagrange multiplier for each constraint row A_i, and we then minimize the unconstrained $\min_p \|(1/2)M^T KM - M^T B + (AM - F)G\|$.

Differentiating with respect to p leads to the augmented system:

$$\begin{bmatrix} K & A^T \\ A & 0 \end{bmatrix} \cdot \begin{bmatrix} M \\ G \end{bmatrix} = \begin{bmatrix} B \\ F \end{bmatrix} \tag{26}$$

solving the earlier linear equations leads to the solution to the constrained system. Note, in this paper, for the sake of saving computational time, only linear constraints are considered. However, it is not difficult to generalize the method to accommodate non-linear constraints by enforcing Lagrange multiplier techniques.

6. Implementation and examples

6.1. Prototype system

A prototype system of diffusion process based turbine blade modeling is implemented on SUN Sparc workstations. Corresponding to the flowchart in Fig. 3, this system includes the following modules: input module, display module, matrix calculation module, and equation solving module.

The input for the system is a B-spline turbine blade, which gives the control points of the volume, and B-spline knot vectors. At each control point, the material value may or may not be known. If it is known, it serves as an initial value. If not, the system can automatically derive it when the user interacts with the system. In this prototype system, each B-spline volume has a second order B-spline basis and has $10 \times 10 \times 10$ control points. The data structure for the

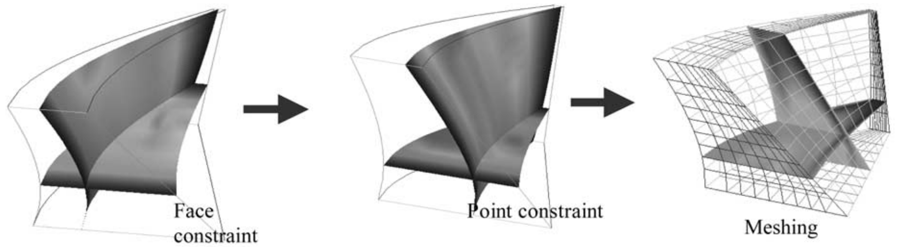

Fig. 7. Material heterogeneity modeling for turbine blade using diffusion equations.

B-spline volume also records the system parameters and imposed constraints.

The display module essentially discretizes the B-spline volume into a field model and connects the output to the Application Visualization System (AVS) [1], which displays the resulting geometry and material heterogeneity.

The matrix calculation module includes the element matrix calculation and assembly processes. When constraints are changed, the system matrices remain the same. Only when the system properties are changed, should the system stiffness matrix and body force matrix be re-calculated. In its implementation, to avoid the ambiguous physical implication of q under the context of HOM, we substitute natural boundary with forced boundary. That is, we only calculate K, B and C (for the time dependent heterogeneous objects).

Gaussian integration is used in the element matrix calculation. For example, assuming the parametric domain of the element is $[u_0, u_1] \times [v_0, v_1] \times [w_0, w_1]$, the K matrix for each element is

$$K_{ij} = \int_{u_0}^{u_1} \int_{v_0}^{v_1} \int_{w_0}^{w_1} D(u,v,w) \left(\frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right) \det(J) \mathrm{d}u\, \mathrm{d}v\, \mathrm{d}w \quad (27)$$

Gaussian integration is used trice to calculate the earlier integration. That is, the earlier equation under some transformation can be changed into the following approximation:

$$\int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} f(\xi, \eta, \zeta) \approx \sum_{i}^{I} \sum_{j}^{J} \sum_{k}^{K} w_i w_j w_k f(\xi_i, \eta_j, \zeta_k)$$

where I, J, K are the number of Gaussian points in u, v and w directions. Given these numbers, we can find Gaussian weights w_i, w_j, w_k and abscissas ξ_i, η_j, ζ_k.

Matrix calculation is a time consuming process. Since all the matrices are symmetric (and positive definite), only half of the elements in the matrices are needed.

All the element matrices are then assembled and turned into a set of linear equations. A equation solver then solves the equations and gives the control point values for the B-spline blade.

6.2. Examples

In the following examples, the diffusion coefficient D is assumed to be uniform and has a unit value. The color variation in the pictures reflects the material composition variation.

6.2.1. Example 1: modeling heterogeneous turbine blades by diffusion equations

Fig. 7 shows one turbine blade modeled by changing system parameters and imposing constraints. In Fig. 7, the B-spline volume undergoes different constraints, which lead to different material heterogeneity distributions. In the left of Fig. 7 two boundary face constraints and one point constraint are imposed. In the right of Fig. 7 these constraints are replaced with a new point constraint. Fig. 7 also includes a meshed model for the turbine blade.

6.2.2. Example 6: heterogeneous turbine blade design

As mentioned in Section 1 the ideal turbine blade is designed to possess the following properties: heat resistance and anti-oxidation properties on the high temperature side, mechanical toughness and strength on the low temperature side, and effective thermal stress relaxation throughout the material.

A heterogeneous turbine blade composed of Al6061 and SiC alloy, is shown in Fig. 8. The thermal conductivities of the two materials are 180 and 25 W/mK. The strengths are $+145/-145$ and $+0/-8300$ Mpa. Using Eqs. (2), (3) and (5), we can have the thermal conductivity and tensile/compression stress for each control point. These properties are, respectively, shown in Fig. 8. Fig. 8 also shows the values at the tip. Note, the notation $a/(b,c)$ in the figure means the value at the tip point is a while the minimal value of the whole volume is b, and the maximum value is c.

Suppose the designer is not satisfied with the strength at the tip of turbine blade, the designer can choose to strengthen the tip by imposing constraints at the tip. The revised model is shown in Fig. 9, where the thermal conductivity has been changed from 25 to 140.94, tensile strength from 0 to 116.92 and compression strength from 8300 to 1724.37.

Using this method, the designer directly interacts with the system using familiar concepts (the material properties)

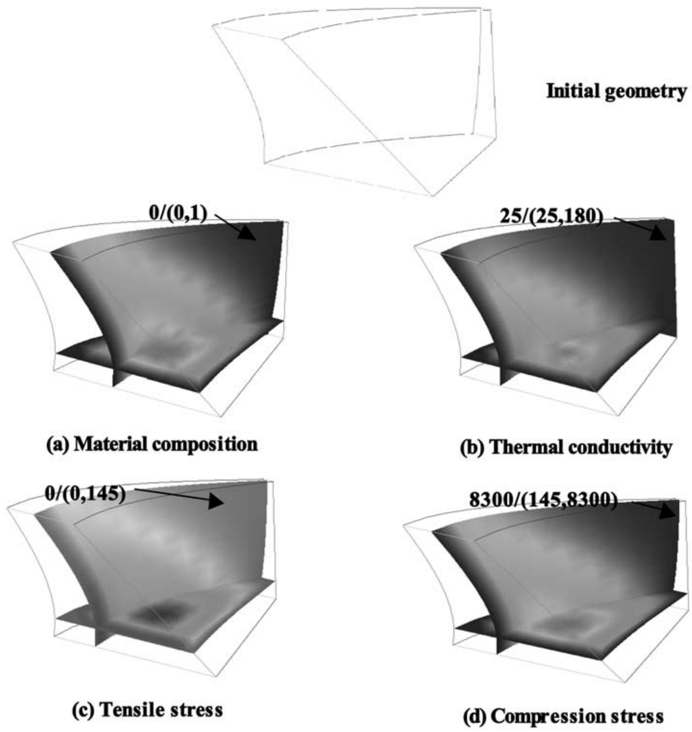

Fig. 8. Turbine blade.

rather than material composition. We believe this direct quantitative feedback of material properties is particularly helpful for designers during the design evolution process.

7. Conclusion

This paper presents a new approach for the design of heterogeneous turbine blade. This approach is based on a physics based modeling method. In this method, the designer guides the design process by specifying the material variation constraints. The designer uses only a few parameters, which have the physical meanings, to control the material composition. During such a control process, the material property variation is directly conceivable to the designer.

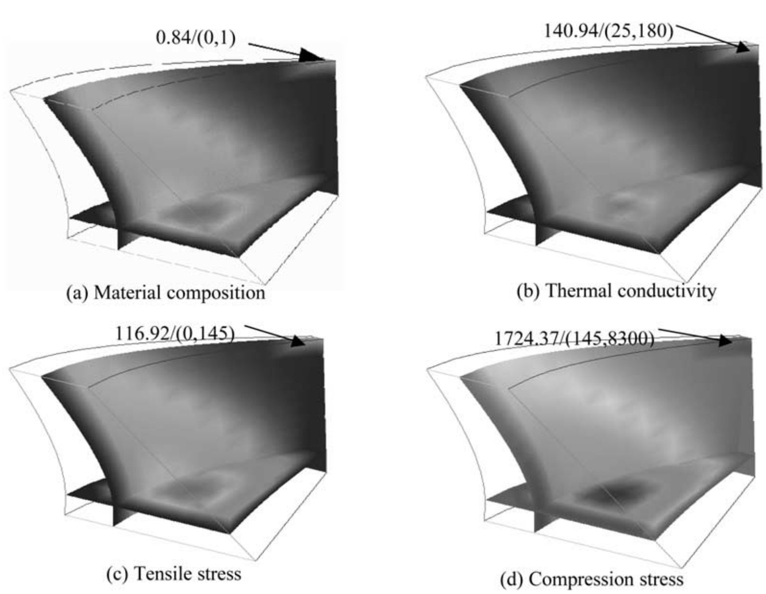

Fig. 9. Turbine blade after modification.

The contribution of this paper is a synthesis of B-spline representation with a diffusion process and FEM. This method employs B-spline as the representation for the turbine blade. It uses finite element method to solve the diffusion equation. Implementation based on this method show that such an approach is an effective way to specify material heterogeneity within turbine blades.

An experienced designer, who understands the diffusion process, would have no problem in using this method for blade design. However, it is possible that the outcome of one diffusion process may not be exactly the same as what the designer wants. In such cases, a few iterations of the diffusion process may be needed. Since the B-spline representation is retained, manual adjustment of a few control points can also be helpful. A more sophisticated approach would be to sub-divide the volume into several sub B-spline volumes (features). Based on this idea, a method called feature based heterogeneous object design has been developed [17].

In this physics based blade design approach, the elements can be naturally derived from the parametric domain of the blades, future work shall integrate this heterogeneous turbine blade modeling method with finite element analysis.

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